Pacific-Basin Finance Journal 77 (2023) 101882

Contents lists available at ScienceDirect

Pacific-Basin Finance Journal

journal homepage: www.elsevier.com/locate/pacfin

Information network, public disclosure and asset prices[☆]

Ronghua Luo[a], Senyang Zhao[b,*], Jing Zhou[c]

[a] Institute of Chinese Financial Studies, Southwestern University of Finance and Economics, Chengdu, 611130, China
[b] School of Finance, Shanghai University of Finance and Economics, and Shanghai Institute of International Finance and Economics, Shanghai, 200433, China
[c] Center for Applied Statistics, School of Statistics, Renmin University of China, Beijing, 100872, China

ARTICLE INFO

Keywords:
Information network
Market quality
Market reaction
Public disclosure

ABSTRACT

We propose in this paper a noisy rational expected equilibrium (NREE) model by taking both public and private information into account with an embedded information network structure among market traders. We derive closed-form expressions for five variables about market reaction and market quality as a function of the topological structure of the network, and we obtain several novel results. First, the information network directly affects the price discovery of private information, but it indirectly influences the price discovery of public information. Second, network connectedness negatively influences both price change and trading volume, while network uniformity only affects trading volume positively and does not impact price change. Third, network connectedness suppressed the marginal effect of public disclosure on market quality, i.e., the information sharing among traders weakens the market quality improvement caused by public disclosure. We extend the NREE model and document a crowding-out effect of information networks in the price discovery of public disclosure. The mechanism of the crowding-out effect also provides a theoretical interpretation of the under-reaction of public disclosure in the market.

1. Introduction

With the rapid development of modern social media technologies and products, such as Weibo in China and Twitter in the U.S., social interaction among investors is becoming increasingly frequent and intensive, which results in a large-scale information network among market traders. This information network changes the operation manners of the stock market by influencing private information sharing among traders and hence asset price. More and more literature explored the important role of information networks and provided many insightful opinions in the past decade. Excellent examples could be found in Ozsoylev and Walden (2011), and Walden (2019). In practice, investors can obtain information from not only private information channels but also public disclosure, which is crucial in the stock market, especially for retail traders. Now that the information network has become vital in traders' decision-making, it naturally raises the following two important questions. First, how does the information network influence the information integration process of public disclosure? Second, how does the information network influence the outcomes of public disclosure, such as the market reaction and market quality changes?

☆ Acknowledgment:
Ronghua Luo's research is supported by Guanghua Talent Project of Southwestern University of Finance and Economics, the National Natural Science Foundation of China (No. 71873110). Jing Zhou's research is supported in part by the National Natural Science Foundation of China (No. 72171226, 11971504), the Beijing Municipal Social Science Foundation (No. 19GLC052) and the National Statistical Science Research Project (No. 2020LZ38).

* Corresponding author.
E-mail addresses: ronghua@swufe.edu.cn (R. Luo), zhaosy@mail.shufe.edu.cn (S. Zhao), zhoujing_89@126.com (J. Zhou).

https://doi.org/10.1016/j.pacfin.2022.101882
Received 18 October 2021; Received in revised form 24 October 2022; Accepted 30 October 2022
Available online 17 November 2022
0927-538X/

To explore the two questions above, we propose a noisy rational expectations equilibrium (NREE) model by taking both public and private information into account with an embedded information network structure among market traders. We get several interesting results by analyzing the proposed NREE model. First, an information network can directly influence the price integration of private information by improving information precision and correlations of different information. In contrast, it can indirectly influence the price integration of public information by passively adjusting investors decision weight of such information.

Second, we propose two network structure measurements and examine their roles in market reaction. They are, respectively, network connectedness and network uniformity. Particularly, network uniformity is a new concept used to describe a network's dispersion. We show that the price change is only affected by network connectedness. The greater the network connectedness is, the smaller the price change will be. However, the trading volume is affected by both connectedness and uniformity. The greater the network connectedness is, the smaller the trading volume will be. The greater the uniformity is, the greater the trading volume will be. It should be noted that network connectedness reflects the average number of neighbors in a network, which is used to decide the average belief in the market. Therefore, network connectedness is highly related to price change since price change usually reflects the average belief change of investors. Furthermore, the trading volume reflects the opinion disagreements in the market, which is consistent with the implication of uniformity.

Third, we examine the influence of network structure on market quality changes due to public disclosure. As we know, market quality reflects the efficiency of price aggregation of information. So a more precise public disclosure always indicates a better market quality. However, the marginal effect of public disclosure on market quality is suppressed by network connectedness. The network brings down the relative importance of public disclosure, decreasing the weight that investors put on public disclosure, which is the network's crowding-out effect in the price integration process of public disclosure. In other words, the information network reduces the marginal effect of public disclosure. This also explains why the price tends to under-react to public disclosure.

The implications of our theoretical model provide guidance for empirical studies that examine the relationship between information networks and market reaction. Specifically, our theoretical analysis demonstrates that the information network may crowd out public disclosure. The underlying mechanism can be tested from the perspective of market reaction. By event study, we can figure out the absolute value of abnormal returns and the abnormal trading volumes caused by public disclosure, which represents the range of price change and trading volume, respectively. We can then construct the information network based on some rules, such as the common shareholding data of mutual funds (Wang et al., 2018; Chen and Li, 2021). It is convenient to test the proposed mechanism empirically by checking the relationship between information networks and market reaction.

Our paper makes the following three contributions. First, we extend the standard NREE model (Hellwig's model) to simultaneously consider public information, private information, and an embedded information network. Consequently, we can investigate how the network affects the price integration process of public disclosure. Second, we extend the structure definition in network analysis and propose a new concept of network uniformity. The variable of network uniformity provides new insight into the role of the network in asset pricing. Finally, the mechanism proposed in this article also helps us understand why prices do not respond well to public disclosure in the stock market, especially in the emerging market.

Our research is meaningful for regulators in emerging markets such as China's stock market. On the one hand, the stock market is heavily influenced by retail investors (Lu et al., 2022), and the information network is more massive and complex compared with the developed market. On the other hand, emerging markets usually suffer from immature trading systems, imperfect information disclosure, low investor literacy, and pricing power in the hands of retail investors. As a result, the market price is more likely to under-react to the public disclosure (Wu, 2013). The existence of information networks may further amplify this underreaction by decreasing the relative importance or the decision weights of public disclosure. Analyzing the relationship between information network structure and the price incorporation process of public disclosure will facilitate a positive response in capital market governance and investor regulation in emerging markets.

The rest of the paper is organized as follows. Section 2 discusses the literature on the NREE model and the relationship between social interaction, pricing, and public disclosure. In Section 3, we introduce the notation used in this paper. We then introduce the model and methods to solve the market equilibrium in Section 4. The main results about the properties of information networks on market reaction and market quality changes are presented in Sections 5 and 6, respectively. Finally, we conclude the paper in Section 7. All the technical proofs are in Appendix.

2. Literatures

Our analysis relies on the NREE model that has been used extensively in the literature on information in financial markets. The NREE model provides a simple but useful method to study the relationship between private information and asset pricing (see, e.g., Hellwig, 1980; Han et al., 2016; Mondria and Yang, 2022). In this framework, investors trade in the market based on their information set, and the CARA-Norm preferences keep investors' demand as a function of price. Then the market-clear condition decides the equilibrium price. The NREE model expresses equilibrium price as the linear weights of different kinds of information. Including an information network and public disclosure may change different kinds of information's correlation and relative precision. Consequently, investors will change the weights they put on different kinds of information when making the trade. This suggests that the information network can influence the price integration of public disclosure.

Our research is also related to the literature on the relationship between social interaction and pricing. Existing studies can be split into two categories. One stream of empirical analyses focused on the impact of the heterogeneous positions of individuals in a network on investors' portfolio returns. For example, Cohen et al. (2008) found that fund managers placed larger bets on firms with whom fund managers share the same education background as firm board members. Trading in "connected" stocks brought in positive abnormal returns compared to trading in "non-connected" stocks. Cici et al. (2017) looked at the intra-family network of fund managers and showed that this type of network also increased investment value. In addition, El-Khatib et al. (2021) constructed the information network among company executives based on their social context. They found that CEO in the core position in a network had better social capital, and his insider trading could gain extra returns. Core position members of the network can profit by explicit mispricing.

The second stream of theory literature focused on how the information network contributed to market aggregate variables, such as trading volume, market liquidity, and price informativeness. For instance, Colla and Mele (2010) assumed that investors shared their information in a cycles-type network. Then, they proved that information linkages raise volume and price informativeness based on the market-neutral pricing rules proposed by Kyle (1985). Ozsoylev and Walden (2011) discussed the same problems under the NREE framework proposed by Hellwig (1980). They found that several aggregate properties of the market were typically non-monotonic functions of network connectedness. However, different from Kyle (1985)'s neutral pricing rules, in emerging markets, such as China, the market traders are all risk-averse and do not need to induce market markers. Thus, our work is more related to Ozsoylev and Walden (2011). Furthermore, Han and Yang (2013) highlight the importance of information acquisition in examining the implications of information networks for financial markets. They found that when information is exogenous, social interaction improves market efficiency. However, social interaction crowd out information production due to traders' incentives to "free ride" on informed friends. As a result, social interaction hurts market efficiency when information is endogenous.

Our research belongs to the second category of the above literature. We focus on the network's role in asset pricing, especially for market aggregate variables caused by public disclosure, such as price change and trading volume. As shown by Goldstein and Yang (2017), people cannot ignore public information in asset pricing for several reasons. First, public information usually consists of the selection of government (Bond and Goldstein, 2015) and publicly-listed firms' disclosures (Gao and Liang, 2013). Both government and firms are the core participants in financial markets. Second, a huge amount of empirical literature has demonstrated that public information improves market quality and facilitates the price discovery process (Tetlock, 2010; Savor, 2012; Landsman et al., 2012; Frenkel et al., 2020). Third, existing researches show that public information may also crowd out the production of private information, which will affect the pricing process and then market quality (Diamond, 1985; Han et al., 2016; Dugast and Foucault, 2018; Kendall, 2018). Goldstein and Yang (2019) studied the effect from the perspective of multiple dimensions of uncertainty. Xue and Zheng (2021) studied the public disclosure choice problems in the capital market. They found that firm improved its disclosure quality when investors were informed with better private signals.

In terms of technical details, our proposed model is most related to previous work done by Kim and Verrecchia (1991), Ozsoylev and Walden (2011), and Han and Yang (2013), but there are three main differences. First, our model is a three-period dynamic model that integrates public information from the market. This setting differs from Ozsoylev and Walden (2011)'s model. Second, a network structure is used to deal with private information, which differs from the one used in Kim and Verrecchia (1991). Third, the network structure used in our paper is more general compared with that in Han and Yang (2013). We can verify that the models of Kim and Verrecchia (1991) and Ozsoylev and Walden (2011) are two special cases of our model. Their conclusions about private information, public information, and welfare also apply to our model. Regarding the research topic, our work is mainly related to the empirical analysis of Wang et al. (2018). They construct the holding-based network of mutual funds to empirically analyze how information diffusion via the network affects the outcomes of public announcements. Their results show that before the earnings announcement, a higher density of information network can help to reduce dis-agreement among funds and thus facilitate the earnings information reflected in stock price. Compared with their work, our work has two main differences. First, they build a network via holding-based data of mutual funds, which is controversial in academics, but our work is pure theoretically and hence can be more general. Second, their conclusions are based on the event study and only set up before the announcement, which reflects the information leakage rather than the network's role in pricing. Our theoretical analysis strengthens the research on the effect of public disclosure.

3. Notation

Unless specified, we always use the following notations: lower case thin letters stand for scalars, upper case thin letters represent sets or functions, lower case bold letters stand for vectors, upper case bold letters represent matrices, and the calligraphy letters represent structures (e.g., $\mathbb{G}_{\mathbb{N}}$ stands for a network).

For a vector $\mathbf{y}$, we define the vector norm as $\|\mathbf{y}\|_p = (\sum_i (\mathbf{y}_i^p))^{1/p}$, and $\|\mathbf{y}\|_\infty = \max_i |(\mathbf{y}_i)|$. The matrix norm is defined as $\|\mathbf{A}\|_p = \sup_{\mathbf{y}:\|\mathbf{y}\|_p=1} \|\mathbf{A}\mathbf{y}\|_p$. We use the notation $diag(\mathbf{d})$ to summarize a diagonal matrix, where $\mathbf{d}$ is a vector. We use T at the right-upper of a matrix or vector to denote the transpose. For convenience, a specific vector is defined as $\mathbf{1}_n^T = (1, 1, \cdots, 1)^T$, or sometimes 1. We write $[\mathbf{A}]_{ij}$ for the scalar in the ith row and jth column of matrix $\mathbf{A}$.

We say that $f(n) = o(g(n))$ if $\lim_{n\to\infty} f(n)/g(n) = 0$, and $f(n) = O(g(n))$ if $\lim_{n\to\infty} f(n)/g(n) = C$, where $0 < C < \infty$ is a constant. If the conditions hold in probability, we say that $f(n) = o_p(g(n))$ and $f(n) = O_p(g(n))$, respectively.

4. The model and market equilibrium

We follow the economic analysis in Kim and Verrecchia (1991), but allow for network relationships between traders in the model. We allow traders to exchange private information about the payoff of the risky asset with their neighbors. This is similar to the model extension from Hellwig (1980) to Ozsoylev and Walden (2011) and Han and Yang (2013), or from Kyle (1985) to Colla and Mele (2010).

4.1. The network setting

We adopt the network settings from Ozsoylev and Walden (2011) with some minor modifications. We use $\mathbb{G}_n = (n, \mathbf{A})$ to summarize a graph with n traders whose relationships are recorded by an adjacency matrix $\mathbf{A}$. The adjacency matrix is in the form $\mathbf{A} = (a_{ij}) \in \mathbb{R}^{n\times n}$, where $a_{ij} = 1$ if trader i is connected with trader j and $a_{ij} = 0$ otherwise ($i \neq j$). Following the conventions, each trader is connected with herself, that is, $a_{ii} = 1$. We use matrix $\mathbf{W} = (w_{ij}) \in \mathbb{R}^{n\times n}$ to represent the number of common neighbors between traders i and j. Based on the definition of $\mathbf{A}$, $\mathbf{W} = \mathbf{A}\mathbf{A}^T$, i.e., $w_{ij} = \sum_{k=1}^{n} a_{ik}a_{jk}$. Particularly, w_{ii} is just the number of neighbors for trader i.

To facilitate the following inference, we first make four assumptions about the network topology as given below[1]:

(i). Traders with more neighbors receive more precise signals;

(ii). All else equal, connected traders have higher signal correlation than non-connected ones;

(iii). If two traders have no common neighbors, then the error terms of their signals are uncorrelated;

(iv). Traders who have the same neighbors receive the same signals.

All assumptions above are mild and quite reasonable. Assumptions (i), (ii) and (iv) can correspond to a linear information structure which is easy to calculate. Especially, assumption (iii) is a traditional NREE condition (Hellwig, 1980) which indicates independent error terms between different traders.

Similar to Ozsoylev and Walden (2011), besides assumptions (i), (ii), (iii) and (iv), we require the information network to have the following two important properties:

$$\|\mathbf{W}_n\|_\infty = o_p(n), \tag{1}$$

$$\lim_{n\to\infty} \frac{\sum_{i=1}^{n} (\mathbf{W}^n)_{ii}}{s^2 n} = b + o_p(1), \tag{2}$$

where s is a bounded positive constant, Eq. (1) means that, for each trader, his/her connected neighbors cannot go to infinity. Equation (1) is shown to be true when the degree distribution is power-low, and b will be an existing constant when the parameter of power-low distribution is larger than 2 (Ozsoylev and Walden, 2011). Eq. (2) suggests a sparse network structure where the number of connections is in the same order as the number of nodes. The constant b represents the network connectedness because it reflects the average connections among all the traders. A larger value of b means a more connective network. Moreover, we define in this paper network uniformity as:

$$\lim_{n\to\infty} \frac{\sum([\mathbf{W}]_{ii} - \frac{[\mathbf{W}]_{ii}}{n})}{n} = u. \tag{3}$$

A smaller value of u means less difference among traders on the network connection, i.e., the network is more uniform among traders.

4.2. The basic economy setting

We consider a pure exchange economy with n traders which are indexed by i, where $i = 1, \cdots, n$. In this economy, all traders face three time periods, referred to as periods 1, 2, and 3. Trading occurs in periods 1 and 2, while consumption happens in period 3. There are two kinds of assets in the market, a risky asset and a riskless bond. One unit of the riskless bond pays off one unit of consumption good in period 3. The return of the risky asset is a normally distributed random variable $\tilde{u}$ with mean $\bar{u}$ and variance h^2, and the return will be realized in period 3.

In period 1, each trader is endowed with z_i risky assets and e_i riskless bonds. The total supply of risky assets is the total endowment of risky assets, which equals $\tilde{z}_{total} = \sum z_i = n\tilde{z}$. It is not known to individual traders, where $\tilde{z} \sim N(\bar{z}, \delta^2)$.[2] All traders in the market observe two kinds of information, one is a public information signal $\tilde{y}_1 = \tilde{u} + \tilde{\varepsilon}$ where $\tilde{\varepsilon} \sim N(0, m_1^2)$, and the other is an initial private information signal $\tilde{\tau}_i = \tilde{u} + \tilde{\epsilon}_i$ where $\tilde{\epsilon}_i \sim N(0, s^2)$. It is assumed that s^2 is uniformly bounded. Particularly, due to the information network, traders can share initial private information with their connected neighbors before the market opens. This kind of information sharing form a new private signal $\tilde{x}_i$ for trader i, which is defined as:

$$\tilde{x}_i = F_i(\tilde{\tau}_1, \ldots, \tilde{\tau}_n | \mathbb{G}_n).$$

According to the assumptions (i), (ii), (iii) and (iv) aforementioned in Section 4.1, we define F_i as a linear information communication rule as:

$$\tilde{x}_i \stackrel{def}{=} \frac{\sum_{k\in R_i} \tilde{\tau}_k}{[\mathbf{W}]_{ii}}.$$

[1] Detail interpretation about the four hypotheses are illustrated by an example in Appendix E.

[2] As pointed out by Grossman (1978), Grossman and Stiglitz (1980), the random supply $\tilde{z}_{total}$ serves as an additional source of uncertainty which prevents securities prices from fully revealing all the private information in the market.

With this linear communication rule, we re-write the new private signal as $\tilde{x}_i = \tilde{u} + \tilde{\eta}_i$, where $\tilde{\eta}_i$ is a normally distributed random variable with mean zero and variance s_i^2. Particularly, the random vector $\tilde{\boldsymbol{\eta}} = (\tilde{\eta}_1, \ldots, \tilde{\eta}_n)$ is multivariate normally distributed with mean zero and covariance matrix $[\mathbf{S}]_{ij} = cov(\tilde{\eta}_i, \tilde{\eta}_j)$, and according to Ozsoylev and Walden (2011), the covariance matrix $\mathbf{S}$ can be decomposed as[3]:

$$\mathbf{S} = s^2\mathbf{D}^{-1}\mathbf{W}\mathbf{D}^{-1}, \tag{4}$$

where $\mathbf{D} = diag([\mathbf{W}]_{11}, \ldots, [\mathbf{W}]_{nn})$. At the end of the period 1, the market opens, and all traders buy and sell securities at competitive market prices based on the information they receive.

At the beginning of the period 2, there is a new public disclosure, that is $\tilde{y}_2 = \tilde{u} + \tilde{v}$, where $\tilde{v} \sim N(0, m_2^2)$. According to the updated information, traders have another round of trading at the end of the period 2. Finally, in period 3, the return of the risky assets is realized, and consumption occurs. As the common setting in NREE literature, we also assume that all random variables are mutually independent.

Traders are all risk-averse, and a negative exponential utility function can represent their performances, i.e., the CARA utility function. Without loss of generality, we assume the risk tolerances for all traders are equal to one,[4] i.e., $U_i(\tilde{\omega}_i) = -\exp(-\tilde{\omega}_i)$. Trader i's final wealth $\tilde{\omega}_i$ can be written as $e_i + \tilde{p}_1 z_i + (\tilde{p}_2 - \tilde{p}_1)\tilde{d}_{1i} + (\tilde{u} - \tilde{p}_2)\tilde{d}_{2i}$, where $\tilde{p}_1$ and $\tilde{p}_2$ are the prices of risky assets in periods 1 and 2, and $\tilde{d}_{1i}$ and $\tilde{d}_{2i}$ are optimal demand of the risky assets in each period. With the incorporation of an information network, traders will update their beliefs about the risky assets from their neighbors. This heterogeneity caused by the network topology is the core analysis in this paper.

4.3. The equilibrium

Traders also know $\tilde{p}_1$ and $\tilde{p}_2$ can reflect the information set held by other traders. Therefore, traders make self-fulfilling conjectures about the relation between prices and trade information in a rational expectation equilibrium. Following Hellwig (1980) and Kim and Verrecchia (1991), we conjecture $\tilde{p}_1$ and $\tilde{p}_2$ can be written as:

$$\tilde{p}_1 = \pi_0 + \pi_1\tilde{y}_1 + \sum \beta_i\tilde{x}_i + \gamma_1\tilde{z}_{total}, \tag{5}$$

$$\tilde{p}_2 = \pi_0' + \pi_1'\tilde{y}_1 + \pi_2'\tilde{y}_2 + \sum \beta_i'\tilde{x}_i + \gamma_2\tilde{z}_{total}. \tag{6}$$

In Eq. (5), $\tilde{p}_1$ is the equilibrium price in period 1, $\tilde{y}_1$ is the public information (e.g. pre-announcements) published in period 1, $\tilde{x}_i$ is the network information that each trader can learn from the market, and $\tilde{z}_{total}$ is the noisy information from noisy traders. The economic implication of the equation is that the equilibrium price in period 1 should be a linear combination of all the available information in the market, which is also an analytic form of effective market hypotheses. The economic implication of Eq. (6) is the same as that in period 1, except that there is a piece of new public information (e.g., announcements $\tilde{y}_2$) in the market. The above equations are different from those in Hellwig (1980), and Kim and Verrecchia (1991) because private information $\tilde{x}_i$ cannot be fully removed due to their mutual correlation.

To simplify the analysis of our model, we further assume that the working mechanism of the information network in pricing is stable across different periods. With this assumption, private information plays the same role in different periods. This suggests that $\beta_i' = k\beta_i$, where k is a positive constant. Then we can easily conjecture that $\tilde{p}_2$ must contains $\tilde{p}_1$, because $\tilde{p}_2$ contains all the available information in $\tilde{p}_1$. The relationship between $\tilde{p}_2$ and $\tilde{p}_1$ is consistent with our intuition that the later pricing process depends on the previous one.

Given the conjectured prices outlined in (5) and (6), all traders choose their optimal demand for the risky assets at the end of periods 1 and 2 by maximizing their utilities. We use a typical reverse recursive method to solve this dynamic programming problem. We first solve the maximizing problem in period 2 and then fold back the results into period 1. In period 2, trader i's available information set consists of the first period public signal $\tilde{y}_1$, the private information $\tilde{x}_i$ which is updated from the information network, the second period public disclosure $\tilde{y}_2$, and the two price signals $\tilde{p}_1$ and $\tilde{p}_2$. The information contained in $\tilde{p}_1$ and $\tilde{p}_2$ is equal to the following signals:

$$\tilde{q}_1 = \frac{\tilde{p}_1 - \pi_0 - \pi_1\tilde{y}_1}{\sum \beta_i} = \tilde{u} + \frac{\sum \beta_i\tilde{\eta}_i}{\sum \beta_i} + \frac{\gamma_1}{\sum \beta_i}\tilde{z}_{total}, \tag{7}$$

$$\tilde{q}_2 = \frac{\tilde{p}_2 - \pi_0' - \pi_1'\tilde{y}_1 - \pi_2'\tilde{y}_2}{\sum \beta_i'} = \tilde{u} + \frac{\sum \beta_i'\tilde{\eta}_i}{\sum \beta_i'} + \frac{\gamma_2}{\sum \beta_i'}\tilde{z}_{total}, \tag{8}$$

where $c_1 = \frac{\gamma_1}{\sum \beta_i}$ and $c_2 = \frac{\gamma_2}{\sum \beta_i'}$. According to Kim and Verrecchia (1991), if $c_1 \neq c_2$, no trading will occur under the situation of fully revealed price information. Thus, we also conjecture that $c_1 = c_2$.[5] This further implies $\tilde{q}_1 = \tilde{q}_2$, i.e., the two price signals are

[3] The detail expanded form of matrix $\mathbf{S}$ is given in Appendix E.

[4] Assuming heterogeneous risk tolerance does not impact the main results of this paper.

[5] It should be noted that Kim and Verrecchia (1991) proved this result without terms $\tilde{\eta}_i$ by the direct usage of large number theory. Based on stable network assumption, the terms $\tilde{\eta}_i$ can be fully removed by subtraction. Therefore our model also meets this conjecture.

perfect substitutes. Let $c_1 = c_2 \equiv c$ and $\tilde{q}_1 = \tilde{q}_2 \equiv \tilde{q}$. The information set in periods 2 and 1 can be transferred to $\{\tilde{y}_1, \tilde{y}_2, \tilde{x}_i, \tilde{q}\}$ and $\{\tilde{y}_1, \tilde{x}_i, \tilde{q}\}$, respectively

Let $\tilde{d}_{2i}$ be the optimal demand of the risky asset for trader i in period 2. Due to the CARA-norm setup on utility function and error terms' multivariate normal distribution, $\tilde{d}_{2i}$ takes a simple expression as:

$$\tilde{d}_{2i} = \frac{(\tilde{\mu}_{2i} - \tilde{p}_2)}{\beta_i^*}, \tag{9}$$

where $\tilde{\mu}_{2i} = E[\tilde{u}|\tilde{y}_1, \tilde{y}_2, \tilde{x}_i, \tilde{q}]$ and $\beta_i^* = \mathrm{var}[\tilde{u}|\tilde{y}_1, \tilde{y}_2, \tilde{x}_i, \tilde{q}]$ are the condition expectation and variance for the payoff of the risky asset when information set is given. Then we can get the risky asset's price at period 2 as described below.

Theorem 1. *Under the market clearing condition $\tilde{z}_{total} = \sum \tilde{d}_{2i}$, as the number of traders in the economy goes to infinity, the equilibrium price in period 2 will converge to*

$$\begin{aligned}\tilde{p}_2 =& \frac{m_1^2 m_2^2 \delta^2 \overline{u} + m_1^2 m_2^2 h^2 b \overline{z}}{k_2} + \frac{h^2 m_2^2 \delta^2}{k_2}\tilde{y}_1 + \frac{h^2 m_1^2 \delta^2}{k_2}\tilde{y}_2 + \\ & \frac{(h^2 m_1^2 m_2^2 \delta^2 b + h^2 m_1^2 m_2^2 b^2)}{k_2}\tilde{u} - \frac{(h^2 m_1^2 m_2^2 \delta^2 + h^2 m_1^2 m_2^2 b)}{k_2}\tilde{z},\end{aligned} \tag{10}$$

where $k_2 = m_2^2 m_1^2 h^2 \delta^2 b + m_1^2 m_2^2 h^2 b^2 + m_1^2 m_2^2 \delta^2 + h^2 m_2^2 \delta^2 + h^2 m_1^2 \delta^2$.

According to the equilibrium condition, we can easily know that (6) and (10) are identical. Therefore, we must have:

$$\begin{aligned}\pi_0' &= \frac{m_1^2 m_2^2 \delta^2 \overline{u} + m_1^2 m_2^2 h^2 b \overline{z}}{k_2}, \\ \pi_1' &= \frac{h^2 m_2^2 \delta^2}{k_2}, \\ \pi_2' &= \frac{h^2 m_1^2 \delta^2}{k_2}, \\ \sum \beta_i' &= \frac{h^2 m_1^2 m_2^2 \delta^2 b + h^2 m_1^2 m_2^2 b^2}{k_2}, \\ \gamma_2 &= -\frac{h^2 m_1^2 m_2^2 \delta^2 + h^2 m_1^2 m_2^2 b}{k_2}.\end{aligned}$$

Then we fold back the result to period 1. In period 1, trader i chooses the optimal demand of the risky asset given signals $\tilde{y}_1$, $\tilde{x}_i$ and $\tilde{p}_1$. Recall that trader i's wealth is $\tilde{w}_i = e_i + \tilde{p}_1 z_i + (\tilde{p}_2 - \tilde{p}_1)\tilde{d}_{1i} + (\tilde{u} - \tilde{p}_2)\tilde{d}_{2i}$ and they also knows the equilibrium price and optimal demand in period 2. Then the maximization problem in period 1 for trader i is to:

$$\begin{aligned}& \max_{\tilde{d}_{1i}} E\left[U_i\left(w_i\right)|\tilde{y}_1, \tilde{x}_i, \tilde{p}_1\right] \\ &= \max_{\tilde{d}_{1i}} E\left[-\exp\left\{e_i + \tilde{p}_1 z_i + (\tilde{p}_2 - \tilde{p}_1)\tilde{d}_{1i} + (\tilde{u} - \tilde{p}_2)\tilde{d}_{2i}\right\}|\tilde{y}_1, \tilde{x}_i, \tilde{q}\right],\end{aligned} \tag{11}$$

subject to (9) and (10).

By solving the maximizing problem above, we get the optimal demand of the risky asset for trader i in period 1 as:

$$\begin{aligned}\tilde{d}_{1i} =& \frac{k_2}{h^2 m_1^2 \delta^2}(m_1^2 \delta^2 \overline{u} + h^2 m_1^2 b \overline{z} + h^2 \delta^2 \tilde{y}_1) + \Bigg[\frac{(h^2 m_1^2 b^2 + h^2 m_1^2 \delta^2 b)k_2}{(h^2 m_1^2 \delta^2)^2} \\ & - b\Bigg]\tilde{q} - \left(\frac{k_{2i} - k_2}{h^2 m_1^2 m_2^2 \delta^2} + \frac{k_2 k_1}{(h^2 m_1^2 \delta^2)^2}\right)\tilde{p}_1,\end{aligned} \tag{12}$$

where:

$$\begin{aligned}k_1 =& m_1^2 h^2 \delta^2 b + m_1^2 h^2 b^2 + m_1^2 \delta^2 + h^2 \delta^2, \\ k_{2i} =& m_1^2 m_2^2 h^2 \delta^2 \frac{1}{s_i^2} + m_1^2 m_2^2 h^2 b^2 + m_1^2 m_2^2 \delta^2 + h^2 m_2^2 \delta^2 + h^2 m_1^2 \delta^2.\end{aligned}$$

Based on this result, we get the equilibrium price in period 1 as presented in the following theorem.

Theorem 2. *Under the market clearing condition $\tilde{z}_{total} = \sum \tilde{d}_{1i}$, as the number of traders in the market goes to infinity, the equilibrium price of the risky asset in period 1 will converge to:*

$$\begin{aligned}\tilde{p}_1 =& \frac{m_1^2 \delta^2 \overline{u} + m_1^2 h^2 b \overline{z}}{k_1} + \frac{h^2 \delta^2}{k_1}\tilde{y}_1 + \frac{(h^2 m_1^2 \delta^2 b + h^2 m_1^2 b^2)}{k_1}\tilde{u} \\ & - \frac{(h^2 m_1^2 \delta^2 + h^2 m_1^2 b)}{k_1}\tilde{z}.\end{aligned} \tag{13}$$

Noting that equilibrium (5) and (13) are identical, we can have:

$$\pi_0 = \frac{m_1^2\delta^2\overline{u} + m_1^2h^2b\overline{z}}{k_1},$$
$$\pi_1 = \frac{h^2\delta^2}{k_1},$$
$$\sum \beta_i = \frac{h^2m_1^2\delta^2b + h^2m_1^2b^2}{k_1},$$
$$\gamma_1 = -\frac{h^2m_1^2\delta^2 + h^2m_1^2b}{k_1}.$$

Remarkably, the proposed model is more general compared to the previous works in Kim and Verrecchia (1991) and Ozsoylev and Walden (2011). We can demonstrate that their models are special cases of ours. See appendix *D* for details.

4.4. Analysis of coefficients of equilibrium prices

According to Theorems 1 and 2, network connectedness plays different roles for different types of signals. Note that $\tilde{u}$ can be entirely substituted by the private information update via communication. Therefore, the coefficient of $\tilde{u}$ reflects how the network affects the price discovery process of private information. Contrastly, the coefficients of $\tilde{y}_1$ and $\tilde{y}_2$ reflect how the network affects the price discovery process of public information. The information communication among network increases the weight of private information while decreasing the weight of public information during the price discovery process. Moreover, b exists in both the molecular and denominator side of the coefficients of $\tilde{u}$, meaning the network has two effects in the price discovery process of private information. The molecular side exhibits the direct effect of the information network. In more detail, due to information communication among network, the overall precision of private information will increase and hence result in a large weight of private information. The denominator side exhibits an indirect effect that reflects the relative importance of different kinds of information. Note that b only exists in the denominator side of the coefficients of $\tilde{y}_1$ and $\tilde{y}_2$, which means the network will decrease the relative importance of public information, resulting in the negative relationship between π_1', π_2' and b.

Note that $\tilde{p}_1$ contains all information ($\tilde{y}_1$, $\tilde{x}_1,\ldots,\tilde{x}_n$, $\tilde{z}$) in period 1 and $\tilde{p}_2$ contains all information ($\tilde{y}_1$, $\tilde{y}_2$, $\tilde{x}_1,\ldots,\tilde{x}_n$, $\tilde{z}$) in period 2. We can write $\tilde{p}_2$ as (see Appendix *D*):

$$\begin{aligned}\tilde{p}_2 &= \alpha^*\tilde{p}_1 + (1-\alpha^*)\tilde{y}_2\\ &= \frac{(b + h^{-2} + m_1^{-2} + b^2\delta^{-2})\tilde{p}_1 + m_2^{-2}\tilde{y}_2}{b + h^{-2} + m_1^{-2} + b^2\delta^{-2} + m_2^{-2}}.\end{aligned}$$

This can be seen as an average weighted function of old information and new information $\tilde{y}_2$. The coefficient $b + h^{-2} + m_1^{-2} + b^2\delta^{-2}$ can be regarded as the precision of $\tilde{p}_1$ and m_2^{-2} is the precision of $\tilde{y}_2$.[6] The equilibrium price in period 2 maintains the same linear structure and scalar as in period 1. The result also verifies the assumption that the network structure is stable across the two periods.

Recall that in the previous section, we analyzed the meaning of the coefficients of the equilibrium price. To better understand the micro mechanism behind equilibrium price, we further analyzed the behavior of investors. Take period 2 as an example, investor i's posterior assessment about the value of risky asset can be expressed as $E[\tilde{u}|\tilde{y}_1, \tilde{y}_2, \tilde{x}_i, \tilde{q}] = \alpha_{0i} + \alpha_{1i}\tilde{y}_1 + \alpha_{2i}\tilde{y}_2 + \alpha_{3i}\tilde{x}_i + \alpha_{4i}\tilde{q}$, where the coefficients α_{1i}, α_{2i}, α_{3i} and α_{4i} can be view as the weights that an investor puts on the corresponding information when they trade. Combine with the concrete expressions in Appendices A and B, we easily have that $\frac{d\alpha_{2i}}{db} < 0$ and $\frac{d\alpha_{4i}}{db} > 0$. With the increase in network connectedness, the investor will put less weight on public information and more on other kinds of information. This indicates that private information is more important than public information. Private information will crowd out public information in the trading process. Such a crowding-out effect might lead to the under-reaction of public information in the pricing process.

5. Market reaction

This section discusses price change, trading volume, and their reaction to network attributes in the large-economy equilibrium characterized by Theorems 1 and 2.

5.1. Price change

Proposition 1. *Under the large-economy equilibrium characterized by Theorems 1 and 2, the price change is:*

$$\begin{aligned}\tilde{p}_2 - \tilde{p}_1 = \frac{h^2m_1^2\delta^2}{k_1k_2}&[m_1^2\delta^2(\tilde{u}-\overline{u}) - h^2\delta^2\tilde{\varepsilon} - m_1^2h^2b\overline{z} + k_1\tilde{v}\\ &+ (h^2m_1^2\delta^2 + h^2m_1^2b)\tilde{z}].\end{aligned} \tag{14}$$

[6] The m_2^{-2} is the precision of error terms. All information in this paper is constituted by the same $\tilde{u}$ plus different error terms. We can use the precision of error terms to represent information precision.

Based on Proposition 1, we immediately get the expected price change as:

$$E|\tilde{p}_2 - \tilde{p}_1| = \frac{(h^2 m_1^2 \delta^2)^2}{k_1 k_2}|\bar{z}| = \frac{\frac{1}{m_2^2}|\bar{z}|}{(b + \frac{b^2}{\delta^2} + \frac{1}{h^2} + \frac{1}{m_1^2})(b + \frac{b^2}{\delta^2} + \frac{1}{h^2} + \frac{1}{m_1^2} + \frac{1}{m_2^2})}. \tag{15}$$

We have the following corollaries according to Eq. (15).

Corollary 5.1. *The expected price change is negatively and convexly correlated with the precision of asset supply (δ^{-2}), and the information precision of initial asset payoff and old public information (h^{-2} and m_1^{-2}). The expected price change is positively and concavely correlated with the information precision of new public disclosure (m_2^{-2}). That is, we always have:*

$$\frac{\partial E(|\tilde{p}_2 - \tilde{p}_1|)}{\partial h^{-2}} < 0, \qquad \frac{\partial E(|\tilde{p}_2 - \tilde{p}_1|)}{\partial m_1^{-2}} < 0,$$
$$\frac{\partial E(|\tilde{p}_2 - \tilde{p}_1|)}{\partial \delta^{-2}} < 0, \qquad \frac{\partial E(|\tilde{p}_2 - \tilde{p}_1|)}{\partial m_2^{-2}} > 0,$$
$$\frac{\partial^2 E(|\tilde{p}_2 - \tilde{p}_1|)}{\partial (h^{-2})^2} > 0, \qquad \frac{\partial^2 E(|\tilde{p}_2 - \tilde{p}_1|)}{\partial (m_1^{-2})^2} > 0,$$
$$\frac{\partial^2 E(|\tilde{p}_2 - \tilde{p}_1|)}{\partial (\delta^{-2})^2} > 0, \qquad \frac{\partial^2 E(|\tilde{p}_2 - \tilde{p}_1|)}{\partial (m_2^{-2})^2} < 0.$$

The results of Corollary 5.1 are intuitive. First, as the precision of disclosure increases, the price change increases. This is because new information helps traders update their beliefs more accurately. However, the speed of this change declines since the precision of the disclosure may not be the most critical factor in deciding the price change when it reaches some level. Some other variables, such as noise, may dominate the price change. Second, except for the precision of disclosure, all other information precisions negatively correlate with the price change and trading volume. This is because all the information is consistent in the two periods. As the information precision increases, the relative importance of public disclosure decreases. Traders will make a few revisions to their beliefs when facing an emerging disclosure. From the second derivative, we can see that the marginal change of this information is monotonically increasing. Holthausen and Verrecchia (1988) showed a similar property for price change in a two-period rational expectation model.

Corollary 5.2. *The expected price change is negatively correlated with the network connectedness (b), while it does not correlate with the network uniformity (u).*

$$\frac{\partial E(|\tilde{p}_2 - \tilde{p}_1|)}{\partial b} < 0, \tag{16}$$
$$\frac{\partial E(|\tilde{p}_2 - \tilde{p}_1|)}{\partial u} = 0. \tag{17}$$

Corollary 5.2 is an exciting result. Recall that we decompose $\tilde{p}_2$ into $\tilde{p}_2 = \alpha^* \tilde{p}_1 + (1 - \alpha^*)\tilde{y}_2$, where α^* represents old information precision weights and (1-α^*) represents precision weights of new information. The absolute price change is related to newly published information. However, the degree of change depends on the relative precision between old and new information. As we know, old information contains private information affected by network structures. Therefore, as network connectedness increases, the relative precision of old information increases. This means $\tilde{p}_2$ is closer to $\tilde{p}_1$, leading to a negative relationship between network connectedness and price change. The price change reflects the average belief change among the market, i.e., it is an aggregate level variable. Thus, it does not correlate with network uniformity.

The negative relationship between the price change and network connectedness also provides a theoretical interpretation of the under-reaction of public disclosure in the market. During the trading process, investors decide how much weight they should put on different information. The more connectedness the network strengthens, the more critical private information is. This motivates investors to put more weight on private information rather than public disclosure, which reflects the under-reaction of public disclosure at the aggregate level.

5.2. Trading volume

Based on Kim and Verrecchia (1991)'s definition, we define the trading volume of trader i on the risky asset as ($s_i^2 = \frac{s^2}{[\mathbf{W}]_{ii}}$):

$$\tilde{d}_{2i} - \tilde{d}_{1i} = -(s_i^{-2} - b)(\tilde{p}_2 - \tilde{p}_1). \tag{18}$$

then we have the following proposition,

Proposition 2. *Under the large-economy equilibrium characterized by Theorems 1 and 2, the total trading volume can be expressed as a summation such as $\frac{1}{2}\sum|\tilde{d}_{2i}-\tilde{d}_{1i}|$, i.e.:*

$$Volume = E[\frac{1}{2}\sum|(s_i^{-2}-b)\;||\;(\tilde{p}_2-\tilde{p}_1)|], \tag{19}$$

$$= \frac{1}{2}\frac{m_2^{-2}s^{-2}\overline{z}nu}{(b+b^2\delta^{-2}+h^{-2}+m_1^{-2})(b+b^2\delta^{-2}+h^{-2}+m_1^{-2}+m_2^{-2})}. \tag{20}$$

Eq. (19) gives a quantitative relationship between price and trading volume, which is consistent with Jain (1988)'s conclusion.

Corollary 5.3. *The relationship between trading volume and information precision is the same as price change.*

This corollary is obvious. Since Eq. (19) contains price change and remaining terms s_i^{-2}, b are independent with other information precisions, e.g. m_1^{-2}, m_2^{-2}. So, the logic behind the price change also holds in the case of the trading volume. However, the terms $|s_i^{-2}-b|$ cause a different reaction of trading volumes with the network.

Corollary 5.4. *The trading volume is negatively correlated with network connectedness (b) while positively correlated with the network uniformity variable (u).*

$$\frac{\partial Volume}{\partial b} < 0, \qquad \frac{\partial Volume}{\partial u} > 0.$$

The effect of network connectedness on trading volume is the same as on price change. It is a natural result because trading volume is positively correlated with the price change, as in Eq. (19). However, unlike the price change, network uniformity also affects trading volume (u). When network connectedness (b) is fixed, we can see that trading volume positively correlates with network uniformity (u). Recall the definition of network uniformity (u), a bigger u suggests a less uniform network. Consequently, a bigger u indicates a more diverse information structure and hence more disagreement among traders, which can facilitate trading.

6. Market quality

In this section, we discuss how market quality is affected by network attributes. Market quality refers to a market's ability to meet its dual goals of liquidity and price discovery (Hara and Ye, 2011). We use three common measures to gauge market quality. The first one is market liquidity which characterizes the market's ability to facilitate the purchase or sale of an asset without drastically affecting the asset's price (Goldstein and Yang, 2017). We use market depth to measure liquidity which is in the same spirit of theory literature as Kyle (1985) and Han et al. (2016). Some empirical research also uses market depth as a proxy variable of liquidity. (e.g.: Ding et al., 2017). The second one is price efficiency, also called market efficiency, which is how informative the prevailing market prices are about the future values of risky assets. In literature (e.g., Vives, 2008; Ozsoylev and Walden, 2011), researchers measure price efficiency by the precision of posterior about fundamental value conditional on its price. The third one is the cost of capital. The expected return $E(\tilde{u}-\tilde{p})$ is often interpreted as the cost of capital on the risky asset (e.g: Easley and Hara, 2004; Lambert et al., 2007), which reflects the uncertainty between prices $\tilde{p}$ and fundamental value $\tilde{u}$.

Based on the above literature, we define price efficiency ($\mathrm{eff}_1, \mathrm{eff}_2$), market liquidity ($\mathrm{liq}_1, \mathrm{liq}_2$) and cost of capital ($\mathrm{cost}_1, \mathrm{cost}_2$) for each period as follows:

$$\mathrm{eff}_1 = \frac{1}{Var(\tilde{u}|\tilde{p}_1)} = h^{-2} + \frac{(m_1^{-2}+b^2\delta^{-2}+b)^2}{m_1^{-2}+b^2\delta^{-2}+2b+\delta^2}, \tag{21}$$

$$\mathrm{eff}_2 = \frac{1}{Var(\tilde{u}|\tilde{p}_2)} = h^{-2} + \frac{(m_1^{-2}+m_2^{-2}+b^2\delta^{-2}+b)^2}{m_1^{-2}+m_2^{-2}+b^2\delta^{-2}+2b+\delta^2}, \tag{22}$$

$$\mathrm{liq}_1 = \frac{1}{\gamma_1} = b + \frac{h^{-2}+m_1^{-2}}{1+b\delta^{-2}}, \tag{23}$$

$$\mathrm{liq}_2 = \frac{1}{\gamma_2} = b + \frac{h^{-2}+m_1^{-2}+m_2^{-2}}{1+b\delta^{-2}}, \tag{24}$$

$$\mathrm{cost}_1 = E(\tilde{u}-\tilde{p}_1) = \frac{\overline{z}}{m_1^{-2}+h^{-2}+b^2\delta^{-2}+b}, \tag{25}$$

$$\mathrm{cost}_2 = E(\tilde{u}-\tilde{p}_2) = \frac{\overline{z}}{m_1^{-2}+m_2^{-2}+h^{-2}+b^2\delta^{-2}+b}. \tag{26}$$

First, we analyze the price efficiency from the perspective of noisy trading. Recall that we have $q = q_1 \equiv q_2$. Han et al. (2016) use ρ (inverse of the variance of $\tilde{q}-\tilde{u}$) to capture the degree of extra information that the price conveys in addition to the public and private information. If the conjecture is right, then the price informativeness should be the same in the two periods. In our model, we have $\rho_{q_1} = \rho_{q_2} = b^{-2}\delta^2$, which means that with the increase of connectedness, the price contains less noisy information. An extreme case of this conclusion is that connectedness goes to infinity. This indicates there is no private information in the market, and then all trade will be conducted by noisy traders. In this case, the price is uninformative.

Corollary 6.1. *The price efficiency is positively correlated with the information precision of public information (m_1^{-2} and m_2^{-2}) and the network connectedness (b).*

$$\frac{\partial\, eff_1}{\partial\, m_1^{-2}} > 0, \qquad \frac{\partial\, eff_2}{\partial\, m_2^{-2}} > 0,$$
$$\frac{\partial\, eff_1}{\partial\, b} > 0, \qquad \frac{\partial\, eff_2}{\partial\, b} > 0.$$

From Corollary 6.1, we know the price efficiency increases if it has more precision from public information. This is because the precision of public information indicates an informative price, which means better price efficiency. With (21) and (22), we can easily obtain the change of price efficiency at the arrival of public disclosure.

$$\Delta \mathrm{eff} = \mathrm{eff}_2 - \mathrm{eff}_1 > 0. \tag{27}$$

It is easy to show that the sign of expression Δeff is always positive. This indicates that public disclosure increases price efficiency, which is consistent with Goldstein and Yang (2017). However, other models predict the opposite effect, i.e., public disclosure harms the price efficiency (e.g., Diamond, 1985; Gao and Liang, 2013) while information acquisition is endogenous. In our model, the information is exogenous; thus, public disclosure always improves price efficiency.

Corollary 6.2. *The market liquidity is positively correlated with the information precision of public information (m_1^{-2} and m_2^{-2}), and has a non-monotone correlation with network connectedness (b). It is positively correlated with the network connectedness (b) when the public information precision (m_1^{-2} and m_2^{-2}) is at a low level, and it is negatively correlated with the network connectedness (b) when the public information precision (m_1^{-2} and m_2^{-2}) is at a high level.*

$$\frac{\partial\, liq_1}{\partial\, information\ precision} > 0, \qquad \frac{\partial\, liq_2}{\partial\, information\ precision} > 0,$$

$$\frac{\partial\, liq_1}{\partial\, b}\begin{cases} > 0, & if \quad b > \sqrt{\delta^2 h^{-2} + \delta^2 m_1^{-2}} - \delta^2, \\ < 0, & if \quad b < \sqrt{\delta^2 h^{-2} + \delta^2 m_1^{-2}} - \delta^2, \end{cases}$$

$$\frac{\partial\, liq_2}{\partial\, b}\begin{cases} > 0, & if \quad b > \sqrt{\delta^2 h^{-2} + \delta^2 m_1^{-2} + \delta^2 m_2^{-2}} - \delta^2, \\ < 0, & if \quad b < \sqrt{\delta^2 h^{-2} + \delta^2 m_1^{-2} + \delta^2 m_2^{-2}} - \delta^2. \end{cases}$$

This result is obvious because more precise information (m_1^{-2}, m_2^{-2}) usually implies less uncertainty about the asset. As a result, changes in liquidity trading are absorbed with a minor price change. By Eq. (23), we have

$$\frac{\partial \mathrm{liq}_1}{\partial b} = 1 - \frac{\delta^{-2}(h^{-2} + m_1^{-2})}{(1 + b\delta^{-2})^2}.$$

If $b > \sqrt{\delta^2 h^{-2} + \delta^2 m_1^{-2}} - \delta^2$, market liquidity increases as the network connectedness increases, otherwise it will decline. Note that the right side of the inequation contains public information precision. When public information precision is at a low level, the market is dominated by private information. The result is consistent with the no public information model of Han and Yang (2013); when it is at a high level, the market is dominated by public information, and it may reverse to inhibition of the role of the network.

From Eqs. (23) and (24), it is easy to show that:

$$\Delta \mathrm{liq} = \mathrm{liq}_2 - \mathrm{liq}_1 = \frac{m_2^{-2}}{1 + b\delta^{-2}} > 0. \tag{28}$$

Based on this formula, we have the following corollary.

Corollary 6.3. *The liquidity change (Δliq) is positively correlated with the public disclosure's precision (m_2^{-2}) and negatively correlated with the network connectedness (b). Their second cross derivative is smaller than* 0.

$$\frac{\partial\, \Delta liq}{\partial\, m_2^{-2}} > 0, \qquad \frac{\partial\, \Delta liq}{\partial\, b} < 0, \qquad \frac{\partial^2\, \Delta liq}{\partial\, m_2^{-2}\, \partial\, b} < 0.$$

On the one hand, the result of Eq. (28) shows that public disclosure can improve market liquidity, and it is consistent with Diamond and Verrecchia (1991). On the other hand, the liquidity change tends to decline as network connectedness increases. This is because the network can only directly affect private information, thus decreasing the liquidity change caused by public information. An extreme case is when network connectedness goes to infinity, all private information in the market is completely transferred to public information, and noisy traders produce all trading. We can also explain this phenomenon using the crowding-out effect. Verrecchia (1982), Diamond (1985), and Han et al. (2016) stated that the disclosure crowds out the product of private information. In our model, agents can share information with their neighbors in the embedded network. The higher the

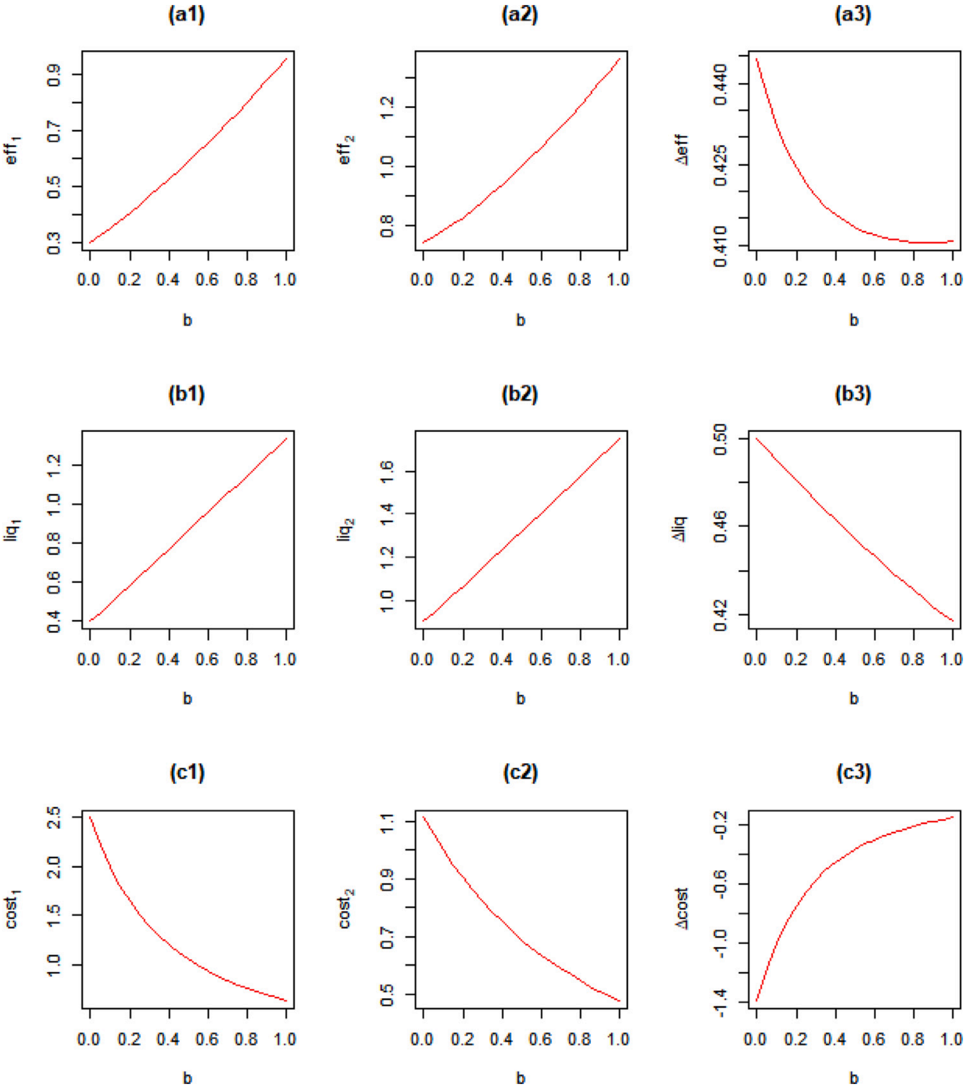

Fig. 1. Numerical Simulation.

connectedness of the network, the more information they get. This reduces the willingness of agents to acquire private information. The negative second cross derivative reflects that public disclosure and the network suppresses mutually in affecting liquidity change. The reason is their contradictory mechanism. Public disclosure ($\tilde{y}_2$) affects the market via the public information channel, while the network affects the market via the private information channel. In the NREE model, it is just the trade-off of the two channels to decide the market equilibrium.

Corollary 6.4. *The cost of capital is negatively correlated with the information precision (m_1^{-2} and m_2^{-2}) and the network connectedness (b).*

$$\frac{\partial\, cost_1}{\partial\, m_1^{-2}} < 0, \qquad \frac{\partial\, cost_2}{\partial\, m_2^{-2}} < 0,$$
$$\frac{\partial\, cost_1}{\partial\, b} < 0, \qquad \frac{\partial\, cost_2}{\partial\, b} < 0.$$

The cost of capital measures the uncertainty between asset price $\tilde{p}$ and fundamental value $\tilde{u}$. A larger m_1^{-2} means more public information, and a larger b indicates more advantage in private information. More information reduces uncertainty about the gaps.

Corollary 6.5. *The absolute value of the cost of capital change ($|\Delta cost|$) is positively correlated with the public disclosure's precision (m_2^{-2}) and negatively correlated with the network connectedness (b). Their second cross derivative is smaller than* 0*.*

$$\frac{\partial\,|\Delta\,cost|}{\partial\,m_2^{-2}} > 0, \qquad \frac{\partial\,|\Delta\,cost|}{\partial\,b} < 0, \qquad \frac{\partial^2\,|\Delta\,cost|}{\partial\,m_2^{-2}\partial\,b} < 0.$$

By straightforward calculation, we have the change in the cost of capital:

$$\Delta cost = \frac{\bar{z}}{m_1^{-2} + m_2^{-2} + h^{-2} + b^2\delta^{-2} + b} - \frac{\bar{z}}{m_1^{-2} + h^{-2} + b^2\delta^{-2} + b} < 0. \tag{29}$$

From Eq. (29), we know that the cost of capital declines if disclosure occurs. The reason is the same as given in Corollary 6.4. A large network connectedness leads to smaller $|\Delta cost|$. More network connectedness means a comparatively less important disclosure, so it suppresses the uncertainty reduction function of disclosure.

Following Goldstein and Yang (2019), we use Fig. 1 to numerically examine the implication of networks in the economy. Without loss of generality, we assume that $m_1^{-2} = h^{-2} = \delta^{-2} = 0.2$. By noting that the precision of disclosure is commonly higher than pre-disclosure, we thus assume $m_2^{-2} = 0.5$. We plot three variables about market quality against network connectedness. The results of numerical examinations are consistent with Corollary 6.1 to 6.5. Figures ($a1$) and ($a2$) indicate that price efficiency is positively related to network connectedness, and Figure ($a3$) indicates that public information disclosure can enhance price efficiency. Note that in our simulation settings, b is always larger than $\sqrt{\delta^2 h^{-2} + \delta^2 m_1^{-2} - \delta^2}$. Thus, Figures ($b1$) and ($b2$) show that liquidity is also positively related to network connectedness. Figure ($b3$) shows that public information disclosure always enhances market liquidity, while the magnitude of liquidity enhancement is negatively related to network connectedness. Similarly, Figures ($c1$) and ($c2$) reflect the relationship between network connectedness and cost of capital. That is, a higher network connectedness will contribute to a lower cost of capital. Figure ($c3$) shows that network connectedness will reduce the magnitude of the cost of capital reduction. Recall that cost of capital is a contrarian indicator of market quality. Thus, Fig. 1 shows that both network connectedness and public disclosure can improve market quality, but network connectedness crowd-outs the market quality improvement caused by public disclosure.

7. Conclusion

We study the role of the information network in price discovery via an NREE model. Our theoretical analysis shows that the information network facilitates the price integration of private information by improving information precision and correlations of various information. At the same time, the information network indirectly influences the price integration of public information by passively adjusting investors' decision weight on public information. Further, we pay particular attention to two network structure measures, network connectedness and network uniformity. We show that the price change is only affected by network connectedness. The greater the network connectedness is, the smaller the price change will be. However, the trading volume is affected by both connectedness and uniformity. The greater the uniformity is, the greater the trading volume will be. Finally, we show that the marginal effect of public disclosure on market quality is suppressed by network connectedness. A numerical study further verifies the conclusion that information networks can decrease the market quality changes caused by public disclosure. Our research suggests that information network and public disclosure helps to improve market quality. However, due to the complementary relationship between the information network and public disclosure in price discovery, the information network will crowd out public information in investors' decision process.

CRediT authorship contribution statement

Ronghua Luo: Conceptualization, Supervision. **Senyang Zhao:** Methodology, Writing – original draft. **Jing Zhou:** Writing – review & editing, Funding acquisition.

Appendix

To facilitate the proof, we decompose the covariance matrix $\mathbf{S}$ into column vectors $\mathbf{S} = [\mathbf{s}_1, \dots, \mathbf{s}_n]$, and we define the scalars $s_i^2 = [\mathbf{S}]_{ii} = \frac{s^2}{[\mathbf{W}]_{ii}}$.

Appendix A. The proof of Theorem 1

Proof. According the model setting, the joint distribution of $(\tilde{u}, \tilde{y}_1, \tilde{y}_2, \tilde{x}_i, \tilde{q})$ in period 2 is a multivariate normal distribution with mean $[\bar{u}, \bar{u}, \bar{u}, \bar{u}, \bar{u} + nc\bar{z}]$ and covariance matrix (where $\boldsymbol{\beta}$ is a column vector $[\beta_1', \beta_2', \beta_n']$)

$$\begin{bmatrix} h^2 & h^2 & h^2 & h^2 & h^2 \\ h^2 & h^2 + m_1^2 & h^2 & h^2 & h^2 \\ h^2 & h^2 & h^2 + m_2^2 & h^2 & h^2 \\ h^2 & h^2 & h^2 & h^2 + \frac{s^2}{[\mathbf{W}]_{ii}} & h^2 + \frac{\boldsymbol{\beta}^T \mathbf{s}_i}{\mathbf{1}^T \boldsymbol{\beta}} \\ h^2 & h^2 & h^2 & h^2 + \frac{\boldsymbol{\beta}^T \mathbf{s}_i}{\mathbf{1}^T \boldsymbol{\beta}} & h^2 + \frac{\boldsymbol{\beta}^T \mathbf{S} \boldsymbol{\beta}}{(\mathbf{1}^T \boldsymbol{\beta})^2} + n^2 c^2 \delta^2 \end{bmatrix}$$

Then by projection theorem, we have that

$$E(\tilde{u}|\tilde{y}_1, \tilde{y}_2, \tilde{x}_i, \tilde{p}_1, \tilde{p}_2) = E(\tilde{u}|\tilde{y}_1, \tilde{y}_2, \tilde{x}_i, \tilde{q}) = \alpha_{0i} + \alpha_{1i}\tilde{y}_1 + \alpha_{2i}\tilde{y}_2 + \alpha_{3i}\tilde{x}_i + \alpha_{4i}\tilde{q}, \tag{30}$$

$$D(\tilde{u}|\tilde{y}_1, \tilde{y}_2, \tilde{x}_i, \tilde{p}_1, \tilde{p}_2) = \beta_i^*, \tag{31}$$

where

$$\alpha_{0i} = \frac{m_1^2 m_2^2 \overline{u}}{b_i}\left[\left(\frac{\boldsymbol{\beta}^T \mathbf{S}\boldsymbol{\beta}}{(\mathbf{1}^T\boldsymbol{\beta})^2} + n^2c^2\delta^2\right)s_i^2 - \left(\frac{\boldsymbol{\beta}^T \mathbf{s}_i}{\mathbf{1}^T\boldsymbol{\beta}}\right)^2\right] + \frac{m_1^2 m_2^2 h^2}{b_i}\left(\frac{\boldsymbol{\beta}^T \mathbf{s}_i}{\mathbf{1}^T\boldsymbol{\beta}} - s_i^2\right)nc\overline{z}, \tag{32}$$

$$\alpha_{1i} = \frac{h^2 m_2^2}{b_i}\left[\left(\frac{\boldsymbol{\beta}^T \mathbf{S}\boldsymbol{\beta}}{(\mathbf{1}^T\boldsymbol{\beta})^2} + n^2c^2\delta^2\right)s_i^2 - \left(\frac{\boldsymbol{\beta}^T \mathbf{s}_i}{\mathbf{1}^T\boldsymbol{\beta}}\right)^2\right], \tag{33}$$

$$\alpha_{2i} = \frac{h^2 m_1^2}{b_i}\left[\left(\frac{\boldsymbol{\beta}^T \mathbf{S}\boldsymbol{\beta}}{(\mathbf{1}^T\boldsymbol{\beta})^2} + n^2c^2\delta^2\right)s_i^2 - \left(\frac{\boldsymbol{\beta}^T \mathbf{s}_i}{\mathbf{1}^T\boldsymbol{\beta}}\right)^2\right], \tag{34}$$

$$\alpha_{3i} = \frac{h^2 m_1^2 m_2^2}{b_i}\left[\frac{\boldsymbol{\beta}^T \mathbf{S}\boldsymbol{\beta}}{(\mathbf{1}^T\boldsymbol{\beta})^2} + n^2c^2\delta^2 - \frac{\boldsymbol{\beta}^T \mathbf{s}_i}{\mathbf{1}^T\boldsymbol{\beta}}\right], \tag{35}$$

$$\alpha_{4i} = \frac{h^2 m_1^2 m_2^2}{b_i}\left[s_i^2 - \frac{\boldsymbol{\beta}^T \mathbf{s}_i}{\mathbf{1}^T\boldsymbol{\beta}}\right], \tag{36}$$

$$\beta_i^* = \frac{h^2 m_1^2 m_2^2}{b_i}\left[\left(\frac{\boldsymbol{\beta}^T \mathbf{S}\boldsymbol{\beta}}{(\mathbf{1}^T\boldsymbol{\beta})^2} + n^2c^2\delta^2\right)s_i^2 - \left(\frac{\boldsymbol{\beta}^T \mathbf{s}_i}{\mathbf{1}^T\boldsymbol{\beta}}\right)^2\right], \tag{37}$$

$$b_i = m_1^2 m_2^2[(h^2 + s_i^2)(h^2 + \frac{\boldsymbol{\beta}^T \mathbf{S}\boldsymbol{\beta}}{(\mathbf{1}^T\boldsymbol{\beta})^2} + n^2c^2\delta^2) - (\frac{\boldsymbol{\beta}^T \mathbf{s}_i}{\mathbf{1}^T\boldsymbol{\beta}} + h^2)^2] + (m_1^2 + m_2^2)h^2[(\frac{\boldsymbol{\beta}^T \mathbf{S}\boldsymbol{\beta}}{(\mathbf{1}^T\boldsymbol{\beta})^2} + n^2c^2\delta^2)s_i^2 - (\frac{\boldsymbol{\beta}^T \mathbf{s}_i}{\mathbf{1}^T\boldsymbol{\beta}})^2]. \tag{38}$$

By taking Eqs. (30) and (31) into Eq. (9) and combining with the market clearing condition, we can get

$$\left(\sum \frac{\alpha_{0i} - \alpha_{4i}\frac{\pi_0'}{k\mathbf{1}^T\boldsymbol{\beta}}}{\beta_i^*}\right) + \left(\sum \frac{\alpha_{1i} - \alpha_{4i}\frac{\pi_1'}{k\mathbf{1}^T\boldsymbol{\beta}}}{\beta_i^*}\right)\tilde{y}_1 + \left(\sum \frac{\alpha_{2i} - \alpha_{4i}\frac{\pi_1'}{k\mathbf{1}^T\boldsymbol{\beta}}}{\beta_i^*}\right)\tilde{y}_2 + \sum \frac{\alpha_{3i}\tilde{x}_i}{\beta_i^*} + \left(\sum \frac{\frac{\alpha_{4i}}{k\mathbf{1}^T\boldsymbol{\beta}} - 1}{\beta_i^*}\right)\tilde{p}_2 = n\tilde{z}. \tag{39}$$

In Eq. (39), we define $\left(\sum \frac{1 - \frac{\alpha_{4i}}{k\mathbf{1}^T\boldsymbol{\beta}}}{\beta_i^*}\right)^{-1} = \gamma$. Then we can solve out the coefficients as:

$$\pi_0' = \frac{\sum \frac{\alpha_{0i}}{\beta_i^*}}{\frac{1}{\gamma} + \frac{\sum \frac{\alpha_{4i}}{\beta_i^*}}{k\mathbf{1}^T\boldsymbol{\beta}}}, \tag{40}$$

$$\pi_1' = \frac{\sum \frac{\alpha_{1i}}{\beta_i^*}}{\frac{1}{\gamma} + \frac{\sum \frac{\alpha_{4i}}{\beta_i^*}}{k\mathbf{1}^T\boldsymbol{\beta}}}, \tag{41}$$

$$\pi_2' = \frac{\sum \frac{\alpha_{2i}}{\beta_i^*}}{\frac{1}{\gamma} + \frac{\sum \frac{\alpha_{4i}}{\beta_i^*}}{k\mathbf{1}^T\boldsymbol{\beta}}}, \tag{42}$$

$$\beta_i' = \gamma\left(\frac{\alpha_{3i}}{\beta_i^*}\right), \tag{43}$$

$$\gamma^* = n\gamma. \tag{44}$$

We further take Eqs. (35), (37) and (38) into Eq. (43) and get

$$\beta_i' = \gamma \frac{\frac{\boldsymbol{\beta}^T \mathbf{S}\boldsymbol{\beta}}{(\mathbf{1}^T\boldsymbol{\beta})^2} + n^2 c^2 \delta^2 - \frac{\boldsymbol{\beta}^T \mathbf{s}_i}{\mathbf{1}^T\boldsymbol{\beta}}}{\left(\frac{\boldsymbol{\beta}^T \mathbf{S}\boldsymbol{\beta}}{(\mathbf{1}^T\boldsymbol{\beta})^2} + n^2 c^2 \delta^2\right) s_i^2 - \left(\frac{\boldsymbol{\beta}^T \mathbf{s}_i}{\mathbf{1}^T\boldsymbol{\beta}}\right)^2} \tag{45}$$

Define $[\mathbf{q}]_i = \frac{\beta_i'}{\gamma}$, Eq. (45) can be simplified as

$$[\mathbf{q}]_i = \frac{1}{s_i^2} \frac{\mathbf{q}^T\mathbf{S}\mathbf{q} + n^2\delta^2 - \mathbf{q}^T\mathbf{s}_i\mathbf{1}^T\mathbf{q}}{\mathbf{q}^T\mathbf{S}\mathbf{q} + n^2\delta^2 - \frac{(\mathbf{q}^T\mathbf{s}_i)^2}{s_i^2}} \tag{46}$$

Define $\mathbf{y} = s^2\mathbf{D}^{-1}\mathbf{q}$ and the vector $\mathbf{d}$ with $[\mathbf{d}]_i = [\mathbf{D}]_{ii}$, we can transform the above formula as

$$[\mathbf{y}]_i = \frac{\mathbf{y}^T\mathbf{W}\mathbf{y} + n^2\delta^2 s^2 - (\mathbf{d}^T\mathbf{y})[\mathbf{d}]_i^{-1}[\mathbf{W}\mathbf{y}]_i}{\mathbf{y}^T\mathbf{W}\mathbf{y} + n^2\delta^2 s^2 - [\mathbf{W}\mathbf{y}]_i^2}, \tag{47}$$

$$[F(\mathbf{y})]_i = 1 + \frac{\frac{[\mathbf{W}\mathbf{y}]_i^2}{n^2} - \frac{(\mathbf{d}^T\mathbf{y}[\mathbf{d}]_i^{-1}[\mathbf{W}\mathbf{y}]_i)}{n^2}}{\frac{(\mathbf{y}^T\mathbf{W}\mathbf{y})}{n^2} + \delta^2 s^2 - \frac{[\mathbf{W}\mathbf{y}]_i^2}{n^2}}. \tag{48}$$

By making use of the same techniques as in Ozsoylev and Walden (2011), for any $\epsilon > 0$ with enough large n, we have

$$\begin{gathered} \mathbf{y} \in \mathbb{R}^n, \quad \|\mathbf{y}\|_\infty <= 2; \\ \Downarrow \\ |[F(\mathbf{y})]_i - 1| <= \frac{\epsilon\delta s^2 + \epsilon\delta s^2}{-\epsilon\delta s^2 + \delta s^2 - \epsilon\delta s^2}. \end{gathered} \tag{49}$$

The Eq. (49) implies that there exists a continuous mapping $F_n : [0,2]^n \to [1-4\epsilon, 1+4\epsilon]^n$, then correspondingly a continuous mapping $F_n : [1-4\epsilon, 1+4\epsilon]^n \to [1-4\epsilon, 1+4\epsilon]^n$. According to the fixed-point theorem, it suggests that there exists some constant $\mathbf{y} \in [1-4\epsilon, 1+4\epsilon]^n$ such that:

$$\lim_{n\to\infty} \|\mathbf{y}_n - \mathbf{1}_n\|_\infty = 0. \tag{50}$$

Define $\mathbf{y} = s^2\mathbf{D}^{-1}\mathbf{q}$, then the equations about $\mathbf{q}$ can be re-expressed as

$$\lim_{n\to\infty} \frac{\mathbf{1}_n^T\mathbf{q}_n}{n} = \lim_{n\to\infty} \frac{\sum [\mathbf{W}]_{ii}[\mathbf{y}]_i}{s^2 n} = b, \tag{51}$$

$$\lim_{n\to\infty} \frac{\mathbf{s}_i^T\mathbf{q}}{n} = 0, \tag{52}$$

$$\lim_{n\to\infty} \frac{\mathbf{q}^T\mathbf{S}\mathbf{q}}{n^2} = 0. \tag{53}$$

As a result, we can further express γ as a function of $\mathbf{q}$

$$\begin{aligned} \frac{1}{\gamma} &= \sum \frac{1 - \frac{\alpha_{4i}}{k\mathbf{1}^T\boldsymbol{\beta}}}{\beta_i^*} = \sum \frac{1 + \frac{h^2 m_1^2 m_2^2}{b_i} \frac{\left(\frac{\boldsymbol{\beta}^T\mathbf{s}_i}{\mathbf{1}^T\boldsymbol{\beta}} - s_i^2\right)}{k\mathbf{1}^T\boldsymbol{\beta}}}{\frac{h^2 m_1^2 m_2^2}{b_i}\left[\left(\frac{\boldsymbol{\beta}^T\mathbf{S}\boldsymbol{\beta}}{(\mathbf{1}^T\boldsymbol{\beta})^2} + \frac{n^2\gamma^2\delta^2}{k^2(\mathbf{1}^T\boldsymbol{\beta})^2}\right) s_i^2 - \left(\frac{\boldsymbol{\beta}^T\mathbf{s}_i}{\mathbf{1}^T\boldsymbol{\beta}}\right)^2\right]} \\ &= \sum \frac{b_i + h^2 m_1^2 m_2^2 \frac{\left(\frac{\boldsymbol{\beta}^T\mathbf{s}_i}{\mathbf{1}^T\boldsymbol{\beta}} - s_i^2\right)}{k\mathbf{1}^T\boldsymbol{\beta}}}{h^2 m_1^2 m_2^2\left[\left(\frac{\boldsymbol{\beta}^T\mathbf{S}\boldsymbol{\beta}}{(\mathbf{1}^T\boldsymbol{\beta})^2} + \frac{n^2\gamma^2\delta^2}{k^2(\mathbf{1}^T\boldsymbol{\beta})^2}\right) s_i^2 - \left(\frac{\boldsymbol{\beta}^T\mathbf{s}_i}{\mathbf{1}^T\boldsymbol{\beta}}\right)^2\right]} \\ &= \sum \frac{\left(\mathbf{1}^T\mathbf{q} - \frac{\mathbf{q}^T\mathbf{s}_i}{s_i^2}\right)^2 - \frac{1}{\gamma}\left(\mathbf{1}^T\mathbf{q} - \frac{\mathbf{q}^T\mathbf{s}_i}{s_i^2}\right)}{\mathbf{q}^T\mathbf{S}\mathbf{q} + n^2\delta^2 - \frac{(\mathbf{q}^T\mathbf{s}_i)^2}{s_i^2}} + \sum \frac{h^2 + s_i^2}{h^2 s_i^2} + \sum \frac{m^2 + n^2}{m^2 n^2}. \end{aligned} \tag{54}$$

Solving the equation above, we have

$$\gamma = \frac{1 + \sum \frac{\mathbf{q}^T\mathbf{1} - \frac{\mathbf{q}^T\mathbf{s}_i}{s_i^2}}{\mathbf{q}^T\mathbf{S}\mathbf{q} + m_2^2\delta^2 - \frac{(\mathbf{q}^T\mathbf{s}_i)^2}{s_i^2}}}{\sum \frac{h^2 + s_i^2}{h^2 s_i^2} + \sum \frac{m_1^2 + m_2^2}{m_1^2 m_2^2} + \sum \frac{\left(\mathbf{1}^T\mathbf{q} - \frac{\mathbf{q}^T\mathbf{s}_i}{s_i^2}\right)^2}{\mathbf{q}^T\mathbf{S}\mathbf{q} + m_2^2\delta^2 - \frac{(\mathbf{q}^T\mathbf{s}_i)^2}{s_i^2}}}. \tag{55}$$

Taking Eqs. (51), (52) and (53) into (55) and multiplying it by n, we can solve γ^* as

$$\begin{aligned}\lim_{n\to\infty}\gamma^* &= \lim_{n\to\infty} n\gamma = \lim_{n\to\infty} n\frac{1+n\frac{bn-0}{0+n^2\delta^2-0}}{n(\frac{1}{h^2}+b+\frac{1}{m_1^2}+\frac{1}{m_2^2}+\frac{(nb)^2}{n^2\delta^2})}\\ &= \frac{1+\frac{b}{\delta^2}}{\frac{1}{h^2}+b+\frac{1}{m_1^2}+\frac{1}{m_2^2}+\frac{b^2}{\delta^2}}\\ &= \frac{h^2m_1^2m_2^2\delta^2+h^2m_1^2m_2^2b}{h^2m_1^2m_2^2\delta^2 b+m_1^2m_2^2\delta^2+h^2m_2^2\delta^2+h^2m_1^2\delta^2+h^2m_1^2m_2^2b^2}\\ &= \frac{h^2m_1^2m_2^2\delta^2+h^2m_1^2m_2^2b}{k_2}.\end{aligned} \tag{56}$$

We further define $\pi^{*\prime} = \lim_{n\to\infty}\boldsymbol{\beta}^T\mathbf{1}$ and have

$$\begin{aligned}(\pi^*)' &= \gamma\sum\frac{\mathbf{q}^T\mathbf{S}\mathbf{q}+n^2\delta 2-(\mathbf{q}^T\mathbf{s}_i)(\mathbf{T}'\mathbf{q})}{\mathbf{q}^T\mathbf{S}\mathbf{q}+n^2\delta^2-\frac{(\mathbf{q}^T\mathbf{s}_i)^2}{s_i^2}\frac{1}{s_i^2}}\\ &= \gamma\sum\frac{n^2\delta^2}{n^2\delta^2}\frac{1}{s_i^2}\\ &= \gamma^*\sum\frac{[\mathbf{W}]_{ii}}{ns^2} = \gamma^* b.\end{aligned} \tag{57}$$

We next show that $\sum\beta_i'\tilde{\eta}_i \xrightarrow{p} 0$. We can firstly note that

$$\begin{aligned}D(\sum\beta_i'\tilde{\eta}_i) &= \boldsymbol{\beta}^T s^2\mathbf{D}^{-1}\mathbf{W}\mathbf{D}^{-1}\boldsymbol{\beta}\\ &= \gamma^2 s^2\mathbf{D}^{-1}\mathbf{q}^T\mathbf{W}\mathbf{q}s^2\mathbf{D}^{-1}\\ &= (\gamma^*)^2\frac{\mathbf{y}^T\mathbf{W}\mathbf{y}}{n^2}\\ &= (\gamma^*)^2\frac{no(n)}{n^2}\xrightarrow{p} 0.\end{aligned}$$

It is easy to show that $E(\sum\beta_i'\tilde{\eta}_i) = 0$, we can thus have $\sum\beta_i'\tilde{\eta}_i \xrightarrow{p} 0$. Then in Eq. (41), we can calculate the molecule and the denominator separately. Further note that $\sum\frac{\alpha_{1i}}{\beta_i^*}\xrightarrow{p}\frac{n}{m_1^2}$ and $\gamma\frac{\sum\frac{\alpha_{4i}}{\beta_i^*}}{k\mathbf{1}^T\boldsymbol{\beta}}\xrightarrow{p}\frac{b}{\delta^2}$, we can get π_1' as

$$\pi_1' \xrightarrow{p} \frac{h^2m_2^2\delta^2}{k_2}. \tag{58}$$

Similarly, we can also get π_2' and π_0'

$$\pi_2' \xrightarrow{p} \frac{h^2m_1^2\delta^2}{k_2}, \tag{59}$$

$$\pi_0' \xrightarrow{p} \frac{m_1^2m_2^2\delta^2\overline{u}+m_1^2m_2^2h^2b\overline{z}}{k_2}. \tag{60}$$

This completes the proof of theorem 1. To simplify the proof of Theorem 2, we further calculate the limitation of α_{0i}, α_{1i}, α_{2i}, α_{3i} and α_{4i}

$$\lim_{n\to\infty}\alpha_{0i} = \frac{m_1^2m_2^2\delta^2\overline{u}+h^2m_1^2m_2^2b\overline{z}}{k_{2i}}, \tag{61}$$

$$\lim_{n\to\infty}\alpha_{1i} = \frac{h^2m_2^2\delta^2}{k_{2i}}, \tag{62}$$

$$\lim_{n\to\infty}\alpha_{2i} = \frac{h^2m_1^2\delta^2}{k_{2i}}, \tag{63}$$

$$\lim_{n\to\infty}\alpha_{3i} = \frac{h^2m_1^2m_2^2\delta^2\frac{1}{s_i^2}}{k_{2i}}, \tag{64}$$

$$\lim_{n\to\infty}\alpha_{4i} = \frac{h^2m_1^2m_2^2b^2}{k_{2i}}, \tag{65}$$

$$\lim_{n\to\infty}\beta_i^* = \frac{h^2m_1^2m_2^2\delta^2}{k_{2i}}, \tag{66}$$

where $k_{2i} = m_1^2 m_2^2 h^2 \delta^2 \frac{1}{s_i^2} + m_1^2 m_2^2 h^2 b^2 + m_1^2 m_2^2 \delta^2 + m_1^2 h^2 \delta^2 + m_2^2 h^2 \delta^2$. □

Appendix B. The proof of Theorem 2

Proof. First, by the same methods as used in the proof of Theorem 1, we can get the following results about the conditional expectations and conditional variance in period 1

$$E[\tilde{u}|\tilde{y}_1, \tilde{x}_i, \tilde{q}] = \tilde{\mu}_{1i} = \theta'_{0i} + \theta'_{1i}\tilde{y}_1 + \theta'_{2i}\tilde{x}_i + \theta'_{3i}\tilde{q} \tag{67}$$

$$\theta'_{0i} \to \frac{\overline{u} m_1^2 \delta^2 + h^2 m_1^2 b \overline{z}}{m_1^2 h^2 \delta^2 \frac{1}{s_i^2} + m_1^2 h^2 b^2 + m_1^2 \delta^2 + h^2 \delta^2}, \tag{68}$$

$$\theta'_{1i} \to \frac{h^2 \delta^2}{m_1^2 h^2 \delta^2 \frac{1}{s_i^2} + m_1^2 h^2 b^2 + m_1^2 \delta^2 + h^2 \delta^2}, \tag{69}$$

$$\theta'_{2i} \to \frac{h^2 m_1^2 \delta^2 \frac{1}{s_i^2}}{m_1^2 h^2 \delta^2 \frac{1}{s_i^2} + m_1^2 h^2 b^2 + m_1^2 \delta^2 + h^2 \delta^2}, \tag{70}$$

$$\theta'_{3i} \to \frac{h^2 m_1^2 b^2 \frac{1}{s_i^2}}{m_1^2 h^2 \delta^2 \frac{1}{s_i^2} + m_1^2 h^2 b^2 + m_1^2 \delta^2 + h^2 \delta^2}. \tag{71}$$

To easy the notation, we define the next two expressions

$$\begin{aligned} k_{1i} &= m_1^2 h^2 \delta^2 \frac{1}{s_i^2} + m_1^2 h^2 b^2 + m_1^2 \delta^2 + h^2 \delta^2, \\ k_1 &= m_1^2 h^2 \delta^2 b + m_1^2 h^2 b^2 + m_1^2 \delta^2 + h^2 \delta^2. \end{aligned}$$

With these two expressions, we have

$$D(\tilde{u}|\tilde{y}_1, \tilde{x}_i, \tilde{q}) \to \frac{h^2 m_1^2 \delta^2}{k_{1i}} \tag{72}$$

In period 1, trader i's information set is $I_1 = \{\tilde{y}_1, \tilde{x}_i, \tilde{p}_1\} = \{\tilde{y}_1, \tilde{x}_i, \tilde{q}\}$, and traders i solves his maximization problem as follows

$$\begin{aligned} &\max_{\tilde{d}_{1i}} E\left[U_i\left(w_i\right)|I_1\right] \\ &\Downarrow \\ &\max_{\tilde{d}_{1i}} E\left[-\exp\{-[e_i + \tilde{p}_1 z_i + (\tilde{p}_2 - \tilde{p}_1)\tilde{d}_{1i} + (\tilde{u} - \tilde{p}_2)\tilde{d}_{2i}]\}|I_1\right] \end{aligned} \tag{73}$$

To simplify the notations, we let $I^* = \{\tilde{y}_1, \tilde{x}_i, \tilde{q}, \tilde{p}_2, \tilde{\mu}_{2i}\}$. By omitting the terms unrelated to $\tilde{d}_{1i}$ and applying the law of iterated expectations for Eq. (73), Eq. (73) can be written as

$$\begin{aligned} &E_{\tilde{p}_2, \tilde{\mu}_{2i}, \tilde{u}}\left[-\exp\left\{(\tilde{p}_1 - \tilde{P}_2)\tilde{d}_{1i} - (\tilde{u} - \tilde{p}_2)\tilde{d}_{2i}\right\}|I_1\right] \\ &= E_{\tilde{p}_2, \tilde{\mu}_{2i}}\left[E_{\tilde{u}}\left[-\exp\left\{(\tilde{p}_1 - \tilde{p}_2)\tilde{d}_{1i} - (\tilde{u} - \tilde{p}_2)\tilde{d}_{2i}\right\}|I^*\right]|I_1\right]. \end{aligned} \tag{74}$$

Noting that the conditional expectation of $\tilde{u}$ in Eq. (74) can be re-written as

$$E[\tilde{u}|I^*] = E[\tilde{u}|\tilde{y}_1, \tilde{x}_i, \tilde{q}, \tilde{p}_2, \tilde{\mu}_{2i}] = E[\tilde{u}|\tilde{y}_1, \tilde{x}_i, \tilde{q}, \tilde{y}_2] = \tilde{\mu}_{2i}.$$

$$\begin{aligned} &E_u\left[-\exp\left\{-(\tilde{u} - \tilde{p}_2)\tilde{d}_{2i}\right\}|I^*\right] \\ =&E_u\left[-\exp\left\{-\frac{(\tilde{u} - \tilde{p}_2)(\tilde{\mu}_{2i} - \tilde{p}_2)}{\beta_i^*}\right\}|\tilde{y}_1, \tilde{x}_i, \tilde{q}, \tilde{p}_2, \tilde{\mu}_{2i}\right] \\ =&-\exp\left\{-\frac{(\tilde{\mu}_{2i} - \tilde{p}_2)^2}{\beta_i^*} + \frac{(\tilde{\mu}_{2i} - \tilde{p}_2)^2 \beta_i^*}{2(\beta_i^*)^2}\right\} \\ =&-\exp\left\{-\frac{(\tilde{\mu}_{2i} - \tilde{p}_2)^2}{2\beta_i^*}\right\}. \end{aligned} \tag{75}$$

According to model setting and results in the proof of Theorem 1, we know that

$$\begin{aligned} &\tilde{\mu}_{2i} - \tilde{p}_2 \\ =&\frac{1}{k_{2i}}\left[k_2\tilde{p}_2 - (h^2 m_1^2 m_2^2 \delta^2 b + h^2 m_1^2 m_2^2 b^2)\tilde{q}\right] - \frac{k_{2i}}{k_{1i}}\tilde{p}_2, \end{aligned}$$

$$=\frac{h^2m_1^2m_2^2\delta^2}{k_{2i}}M, \tag{76}$$

where $M=(b-\frac{1}{s_i^2})\tilde{p}_2+\frac{1}{s_i^2}\tilde{x}_i-b\tilde{q}$. Then by taking Eqs. (75), (76) into (74), Eq. (74) can be re-expressed as

$$E_{\tilde{p}_2,\tilde{\mu}_{2i}}\left[-\exp\left\{(\tilde{p}_1-\tilde{p}_2)\tilde{d}_{1i}-\frac{(\tilde{\mu}_{2i}-\tilde{p}_2)^2}{2\beta_i^*}\right\}|I_1\right]$$
$$=E_{\tilde{p}_2}\left[-\exp\left\{(\tilde{p}_1-\tilde{p}_2)\tilde{d}_{1i}-\frac{h^2m_1^2m_2^2\delta^2}{2k_{2i}}M^2\right\}|I_1\right]. \tag{77}$$

To solve the optimal demand function in period 1 in Eq. (77), we need to attain $E[\tilde{p}_2|\tilde{y}_1,\tilde{x}_i,\tilde{q}]$ and the $D(\tilde{p}_2|\tilde{y}_1,\tilde{x}_i,\tilde{q})$ respectively

$$E[p_2|\tilde{y}_1,\tilde{x}_i,\tilde{q}]=\frac{1}{k_2}\left[m_1^2m_2^2\delta^2\overline{u}+h^2m_1^2m_2^2b\overline{z}+h^2m_2^2\delta^2\tilde{y}_1\right]$$
$$+\frac{h^2m_1^2\delta^2}{k_2}\tilde{\mu}_{1i}+\frac{h^2m_1^2m_2^2b+h^2m_1^2m_2^2b^2}{k_2}\tilde{q}, \tag{78}$$

$$E[p_2|\tilde{y}_1,\tilde{x}_i,\tilde{q}]=\frac{1}{k_2k_{1i}}\left[k_{2i}(m_1^2\delta^2\overline{u}+h^2m_1^2b\overline{z}+h^2\delta^2\tilde{y}_1)\right.$$
$$\left.+h^2m_1^2\delta^2h^2m_1^2\delta^2\frac{1}{s_i^2}\tilde{x}_i+(h^2m_1^2b^2k_{2i}+h^2m_1^2m_2^2\delta^2bk_{1i})\tilde{q}\right] \tag{79}$$

$$D\left(\tilde{p}_2|\tilde{y}_1,\tilde{x}_i,\tilde{q}\right)=D\left(\frac{h^2m_1^2\delta^2(\tilde{u}+\tilde{v})}{k_2}|\tilde{y}_1,\tilde{x}_i,\tilde{q}\right)$$
$$=\frac{(h^2m_1^2\delta^2)^2k_{2i}}{k_{1i}k_2^2} \tag{80}$$

To simplify the notations, we define

$$F=k_{2i}(m_1^2\delta^2\overline{u}+h^2m_1^2b\overline{z}+h^2\delta^2\tilde{y}_1)+(h^2m_1^2\delta^2)^2\frac{1}{s_i^2}\tilde{x}_i$$
$$+(h^2m_1^2b^2k_{2i}+h^2m_1^2m_2^2\delta^2bk_{1i})\tilde{q}, \tag{81}$$

$$E_{\tilde{p}_2}\left[-\exp\left\{(\tilde{p}_1-\tilde{p}_2)\tilde{d}_{1i}-\frac{h^2m_1^2m_2^2\delta^2}{2k_{2i}}M^2\right\}|I_1\right]$$
$$\propto-\int\exp\left[-\frac{1}{2}\left\{-2(\tilde{p}_1-\tilde{p}_2)\tilde{d}_{1i}+\frac{h^2m_1^2m_2^2\delta^2}{k_{2i}}M^2\right.\right.$$
$$\left.\left.+\frac{k_{1i}k_2^2}{(h^2m_1^2\delta^2)^2k_{2i}}\left[\tilde{p}_2-\frac{1}{k_2k_{1i}}F\right]^2\right\}\right]d\tilde{p}_2. \tag{82}$$

We tidy Eq. (82) as follows

$$-\int\exp\left[-\frac{1}{2}\left\{(\frac{h^2m_1^2m_2^2\delta^2}{k_{2i}}(b-\frac{1}{s_i^2})^2+\frac{k_{1i}k_2^2}{(h^2m_1^2\delta^2)^2k_{2i}})\tilde{p}_2^2-2\tilde{p}_2\right.\right.$$
$$\left.\left(\frac{h^2m_1^2m_2^2\delta^2}{k_{2i}}(\frac{1}{s_i^2}\tilde{x}_i-b\tilde{q})(\frac{1}{s_i^2}-b)+\frac{k_2}{(h^2m_1^2\delta^2)^2k_{2i}}F-\tilde{d}_{1i}\right)\right\}$$
$$\left.+\tilde{p}_1\tilde{d}_{1i}\right]d\tilde{p}_2, \tag{83}$$

where

$$\frac{h^2m_1^2m_2^2\delta^2}{k_{2i}}(b-\frac{1}{s_i^2})^2+\frac{k_{1i}k_2^2}{(h^2m_1^2\delta^2)^2k_{2i}}$$
$$=\frac{(k_2-k_{2i})^2}{h^2m_1^2m_2^2\delta^2k_{2i}}+\frac{n^4k_{1i}k_2^2}{(h^2m_1^2m_2^2\delta^2)^2k_{2i}}$$
$$=\frac{1}{(h^2m_1^2m_2^2\delta^2)^2k_{2i}}\left[m_2^2k_2^2k_{2i}-2h^2m_1^2m_2^2\delta^2k_2k_{2i}+h^2m_1^2m_2^2\delta^2k_{2i}^2\right]$$
$$=\frac{k_{2i}-k_2}{h^2m_1^2m_2^2\delta^2}+\frac{m_2^2k_2^2-h^2m_1^2m_2^2\delta^2k_2}{(h^2m_1^2m_2^2\delta^2)^2}$$

$$=\frac{k_{2i}-k_2}{h^2m_1^2m_2^2\delta^2}+\frac{k_2k_1}{(h^2m_1^2\delta^2)^2}, \tag{84}$$

and

$$\frac{h^2m_1^2m_2^2\delta^2}{k_{2i}}\left(\frac{1}{s_i^2}\tilde{x}_i-b\tilde{q}\right)\left(\frac{1}{s_i^2}-b\right)+\frac{k_2}{(h^2m_1^2\delta^2)^2k_{2i}}F$$
$$=\frac{k_2}{(h^2m_1^2\delta^2)^2}(m_1^2\delta^2\overline{u}+h^2m_1^2b\overline{z}+h^2\delta^2\tilde{y}_1)+\frac{1}{s_i^2}\tilde{x}_i$$
$$+\left[\frac{(h^2m_1^2b^2+h^2m_1^2\delta^2b)k_2}{(h^2m_1^2\delta^2)^2}-b\right]\tilde{q}. \tag{85}$$

Then by taking Eqs. (84) and (85) into Eq. (83), and omitting the terms unrelated to $\tilde{d}_{1i}$ and $\tilde{p}_2$, we have

$$-\int \exp\Bigg[-\frac{1}{2}\Bigg\{\left(\frac{k_{2i}-k_2}{h^2m_1^2m_2^2\delta^2}+\frac{k_1k_2}{(h^2m_1^2\delta^2)^2}\right)\tilde{p}_2^2-2\tilde{p}_2$$
$$\Bigg[\frac{k_2}{(h^2m_1^2\delta^2)^2}(m_1^2\delta^2\overline{u}+h^2m_1^2b\overline{z}+h^2\delta^2\tilde{y}_1)+\frac{1}{s_i^2}\tilde{x}_i$$
$$+\left\{\frac{(h^2m_1^2b^2+h^2m_1^2\delta^2A)k_2}{(h^2m_1^2\delta^2)^2}-b\right\}\tilde{q}-\tilde{d}_{1i}\Bigg]\Bigg\}+\tilde{p}_1\tilde{d}_{1i}\Bigg]d\tilde{p}_2$$

The above integral can be written as

$$-exp[\tilde{p}_1\tilde{d}_{1i}$$
$$+\frac{\left\{\frac{k_2}{(h^2m_1^2\delta^2)^2}(m_1^2\delta^2\overline{u}+h^2m_1^2b\overline{z}+h^2\delta^2\tilde{y}_1)+\frac{1}{s_i^2}\tilde{x}_i+\left[\frac{(h^2m_1^2b^2+h^2m_1^2\delta^2b)k_2}{(h^2m_1^2\delta^2)^2}-b\right]\tilde{q}-\tilde{d}_{1i}\right\}^2}{2\left(\frac{k_{2i}-k_2}{h^2m_1^2m_2^2\delta^2}+\frac{k_1k_2}{(h^2m_1^2\delta^2)^2}\right)}]$$

$$\int exp\left[-\frac{1}{2}\left(\frac{k_{2i}-k_2}{h^2m_1^2m_2^2\delta^2}+\frac{k_1k_2}{(h^2m_1^2\delta^2)^2}\right)\Bigg[\tilde{p}_2\right.$$
$$-\Bigg(\frac{k_2}{(h^2m_1^2\delta^2)^2}(m_1^2\delta^2\overline{u}+h^2m_1^2b\overline{z}+h^2\delta^2\tilde{y}_1)+\frac{1}{s_i^2}\tilde{x}_i$$
$$\left.+\left[\frac{(h^2m_1^2b^2+h^2m_1^2\delta^2b)k_2}{(h^2m_1^2\delta^2)^2}-b\right]\tilde{q}-\tilde{d}_{1i}\Bigg)\Bigg/\left(\frac{k_{2i}-k_2}{h^2m_1^2m_2^2\delta^2}+\frac{k_1k_2}{(h^2m_1^2\delta^2)^2}\right)\Bigg]^2\right]d\tilde{p}_2.$$

Noting that the integral part in the above expression contains a core of normal density with mean:

$$\frac{\frac{k_2}{(h^2m_1^2\delta^2)^2}(m_1^2\delta^2\overline{u}+h^2m_1^2b\overline{z}+h^2\delta^2\tilde{y}_1)+\frac{1}{s_i^2}\tilde{x}_i+\left[\frac{(h^2m_1^2b^2+h^2m_1^2\delta^2b)k_2}{(h^2m_1^2\delta^2)^2}-b\right]\tilde{q}-\tilde{d}_{1i}}{\left(\frac{k_{2i}-k_2}{h^2m_1^2m_2^2\delta^2}+\frac{k_1k_2}{(h^2m_1^2\delta^2)^2}\right)}$$

and variance $\left[\frac{k_{2i}-k_2}{h^2m_1^2m_2^2\delta^2}+\frac{k_1k_2}{(h^2m_1^2\delta^2)^2}\right]^{-1}$. Which suggests that the integral can be ignored and we only need to maximize the exponent part in the above expression. Thus, we can attain the demand function of period 1 as

$$\tilde{d}_{1i}=\frac{k_2}{(h^2m_1^2\delta^2)}(m_1^2\delta^2\overline{u}+h^2m_1^2b\overline{z}+h^2\delta^2\tilde{y}_1)$$
$$+\left[\frac{(h^2m_1^2b^2+h^2m_1^2\delta^2b)k_2}{(h^2m_1^2\delta^2)^2}-b\right]\tilde{q}$$
$$-\left(\frac{k_{2i}-k_2}{h^2m_1^2m_2^2\delta^2}+\frac{k_2k_1}{(h^2m_1^2\delta^2)^2}\right)\tilde{p}_1.$$

Next, by the market clear condition, $\sum\tilde{d}_{1i}=n\tilde{z}$, we have

$$n\tilde{z}=\sum\frac{k_2m_1^2\delta^2\overline{u}}{(h^2m_1^2\delta^2)^2}+\sum\frac{k_2h^2m_1^2b\overline{z}}{(h^2m_1^2\delta^2)^2}+\sum\frac{k_2h^2\delta^2\tilde{y}_1}{(h^2m_1^2\delta^2)^2}+\sum\frac{1}{s_i^2}\tilde{x}_i$$
$$+\sum\left\{\frac{(h^2m_1^2b^2+h^2m_1^2\delta^2b)k_2}{(h^2m_1^2\delta^2)^2}-b\right\}\tilde{q}$$

$$- \sum \left(\frac{k_{2i} - k_2}{h^2 m_1^2 m_2^2 \delta^2} + \frac{k_1 k_2}{(h^2 m_1^2 \delta^2)^2} \right) \tilde{p}_1.$$

Taking the limitation form of $\tilde{q} = \tilde{u} - \frac{1}{b}\tilde{z}$ into above expression, we can solve out optimization problem in period 1 and have that

$$\pi_0 = \frac{m^2\delta^2\overline{u} + h^2 m_1^2 b\overline{z}}{k_1}, \tag{86}$$

$$\pi_1 = \frac{h^2\delta^2}{k_1}, \tag{87}$$

$$\pi^{**} = \frac{h^2 m_1^2 b^2 + h^2 m_1^2 \delta^2 b}{k_1}, \tag{88}$$

$$\gamma^{**} = \frac{h^2 m_1^2 b + h^2 m_1^2 \delta^2}{k_1}. \tag{89}$$

This completes the proof of Theorem 2. □

Appendix C. The proof of propositions and corollaries

In this section, we provide the proof of proposition 2, and equation 21. Other proof processes are easy to handle, so we omit them.

Based on the results in Theorem 1 and 2, as a natural result, we have:

$$E(|\tilde{p}_2 - \tilde{p}_1|) = \frac{(h^2 m_1^2 \delta^2)^2}{k_1 k_2} |\overline{z}|.$$

Proof of Proposition 2. We calculate each period's demand for trader i:

$$\begin{aligned}\tilde{d}_{1i} = {}& \frac{1}{s_i^2}\tilde{\eta}_i + \frac{\frac{1}{s_i^2} - b}{k_1}[m_1^2\delta^2(\tilde{u} - \overline{u}) - m_1^2 h^2 b\overline{z} - h^2\delta^2\tilde{\epsilon} \\ & + (h^2 m_1^2 \delta^2 + h^2 m_1^2 b)\tilde{z}] + \tilde{z},\end{aligned} \tag{90}$$

$$\begin{aligned}\tilde{d}_{2i} = {}& \frac{1}{s_i^2}\tilde{\eta}_i + \frac{m_2^2(\frac{1}{s_i^2} - b)}{k_2}[m_1^2\delta^2(\tilde{u} - \overline{u}) - m_1^2 h^2 b\overline{z} - h^2\delta^2\tilde{\epsilon} \\ & - \frac{m_1^2 h^2 \delta^2}{m_2^2}\tilde{\nu} + (h^2 m_1^2 \delta^2 + h^2 m_1^2 b)\tilde{z}] + \tilde{z}.\end{aligned} \tag{91}$$

So the demand change of trader i is:

$$\begin{aligned}\tilde{d}_{2i} - \tilde{d}_{1i} = {}& (\frac{m_2^2}{k_2} - \frac{1}{k_1})(\frac{1}{s_i^2} - b)[m_1^2\delta^2(\tilde{u} - \overline{u}) - m_1^2 h^2 b\overline{z} - h^2\delta^2\tilde{\epsilon} \\ & + (h^2 m_1^2 \delta^2 + h^2 m_1^2 b)\tilde{z}] - \frac{m_1^2 h^2 \delta^2(\frac{1}{s_i^2} - b)}{k_2}\tilde{\nu} \\ = {}& \frac{-m_1^2 h^2 \delta^2(\frac{1}{s_i^2} - b)}{k_1 k_2}[(\tilde{p}_2 - \tilde{p}_1)\frac{k_2 k_1}{h^2 m_1^2 \delta^2} - k_1\tilde{\mu}] \\ & - \frac{m_1^2 h^2 \delta^2(\frac{1}{s_i^2} - b)}{k_2}\tilde{\mu} \\ = {}& -(\frac{1}{s_i^2} - b)(\tilde{p}_2 - \tilde{p}_1) \quad \square\end{aligned} \tag{92}$$

Proof of Eq. 21. Here we provide the calculation process of eff_1, the eff_2 is similar with eff_1. based on the proof process of Theorem 2, $\tilde{p}_1 = \pi_0 + \pi_1\tilde{y}_1 + \pi^{**}\tilde{u} + \gamma^{**}\tilde{z}$

$$\begin{aligned} var(\tilde{p}_1) &= \pi_1^2(h^2 + m_1^2) + (\pi^{**})^2 h^2 + (\gamma^{**})^2\delta^2 + 2\pi_1\pi^{**}h^2 \\ cov(\tilde{p}_1, \tilde{u}) &= \pi_1 h^2 + \pi^{**}h^2 \\ var(\tilde{u}) &= h^2 \end{aligned}$$

then

$$\mathrm{var}(\tilde{u}|\tilde{p}_1) = var(\tilde{u}) - \frac{cov(\tilde{p}_1, \tilde{u})^2}{var(\tilde{p}_1)}$$

bring the above values into $\mathrm{var}(\tilde{u}|\tilde{p}_1)$ and then take inverse, we can receive Eq. (21) □

Appendix D. The relationship with existing models

We discuss in this subsection the relationship between our model and the existing related models by Kim and Verrecchia (1991) and Ozsoylev and Walden (2011). As aforementioned, our proposed model is more general. To illustrate it, we rewrite the equilibrium price in the information precision form as:

$$\begin{aligned}\tilde{p}_1 = &\frac{\frac{1}{h^2}\tilde{u} + \frac{1}{\delta^2}b\tilde{z}}{b + \frac{1}{h^2} + \frac{1}{m_1^2} + b^2\frac{1}{\delta^2}} + \frac{\frac{1}{m_1^2}}{b + \frac{1}{h^2} + \frac{1}{m_1^2} + b^2\frac{1}{\delta^2}}\tilde{y}_1 \\ &+ \frac{b + b^2\frac{1}{\delta^2}}{b + \frac{1}{h^2} + \frac{1}{m_1^2} + b^2\frac{1}{\delta^2}}\tilde{u} - \frac{1 + b\frac{1}{\delta^2}}{b + \frac{1}{h^2} + \frac{1}{m_1^2} + b^2\frac{1}{\delta^2}}\tilde{z},\end{aligned} \tag{93}$$

$$\begin{aligned}\tilde{p}_2 = &\frac{\frac{1}{h^2}\tilde{u} + \frac{1}{\delta^2}b\tilde{z}}{b + \frac{1}{h^2} + \frac{1}{m_1^2} + \frac{1}{m_2^2} + b^2\frac{1}{\delta^2}} + \frac{\frac{1}{m_1^2}}{b + \frac{1}{h^2} + \frac{1}{m_1^2} + \frac{1}{m_2^2} + b^2\frac{1}{\delta^2}}\tilde{y}_1 \\ &+ \frac{\frac{1}{m_2^2}}{b + \frac{1}{h^2} + \frac{1}{m_1^2} + \frac{1}{m_2^2} + b^2\frac{1}{\delta^2}}\tilde{y}_2 + \frac{b + b^2\frac{1}{\delta^2}}{b + \frac{1}{h^2} + \frac{1}{m_1^2} + \frac{1}{m_2^2} + b^2\frac{1}{\delta^2}}\tilde{u} \\ &- \frac{1 + b\frac{1}{\delta^2}}{b + \frac{1}{h^2} + \frac{1}{m_1^2} + \frac{1}{m_2^2} + b^2\frac{1}{\delta^2}}\tilde{z}.\end{aligned} \tag{94}$$

The model of Ozsoylev and Walden (2011) has only one period and has no public information signal. The equilibrium price in their model is:

$$\tilde{p} = \frac{\frac{1}{h^2}\bar{u} + \frac{1}{\delta^2}b\bar{z}}{b + \frac{1}{h^2} + b^2\frac{1}{\delta^2}} + \frac{b + b^2\frac{1}{\delta^2}}{b + \frac{1}{h^2} + b^2\frac{1}{\delta^2}}\tilde{u} - \frac{1 + b\frac{1}{\delta^2}}{b + \frac{1}{h^2} + b^2\frac{1}{\delta^2}}\tilde{z}, \tag{95}$$

where the notation is as same as our model. By noting that there is no public information signal, which means $\frac{1}{m_1^2} = \frac{1}{m_2^2} = 0$. Taking the restriction into (93) and (94), we can easily see that (93) and (94) collapse to (95). This shows that Ozsoylev and Walden (2011)'s model is really a special case of our model.

We then show that Kim and Verrecchia (1991)'s model is also a special case of our model. Different from our model, Kim and Verrecchia (1991) consider a continuous trader system where traders have independent error terms (i.e., the traders in the market do not share their private information.) By making unified notation, the equilibrium price they obtain is:

$$\begin{aligned}\tilde{p}_1 \;=\; &\frac{1}{\frac{1}{h^2} + \frac{1}{m_1^2} + \frac{1}{s^2} + (\frac{1}{s^2})^2\frac{1}{\delta^2}}[\frac{1}{h^2}\bar{u} + \frac{1}{m_1^2}\tilde{y}_1 + (\frac{1}{s^2} + (\frac{1}{s^2})^2\frac{1}{\delta^2})\tilde{u} \\ &-(1 + \frac{1}{s^2}\frac{1}{\delta^2})\tilde{z}],\end{aligned} \tag{96}$$

$$\begin{aligned}\tilde{p}_2 \;=\; &\frac{1}{\frac{1}{h^2} + \frac{1}{m_1^2} + \frac{1}{m_2^2} + \frac{1}{s^2} + (\frac{1}{s^2})^2\frac{1}{\delta^2}}[\frac{1}{h^2}\bar{u} + \frac{1}{m_1^2}\tilde{y}_1 + \frac{1}{m_2^2}\tilde{y}_2 + (\frac{1}{s^2} \\ &+(\frac{1}{s^2})^2\frac{1}{\delta^2})\tilde{u} - (1 + \frac{1}{s^2}\frac{1}{\delta^2})\tilde{z}].\end{aligned} \tag{97}$$

As mentioned previously, $b = \frac{\sum[\mathbf{W}]_{ii}}{s^2 n}$ in our model represents the connectedness of the information network. The absence of an information network means that each trader only connects with herself and all traders have independent private signals. This suggests that $\sum[\mathbf{W}]_{ii} = n$, then $b = \frac{1}{s^2}$, and hence Eqs. (93) and (94) just become (96) and (97). This demonstrates that the model of Kim and Verrecchia (1991) is a special case of our model.

Appendix E. An example about the network

We give an example in Fig. E.1 to better illustrate the economic implications of the four hypotheses.

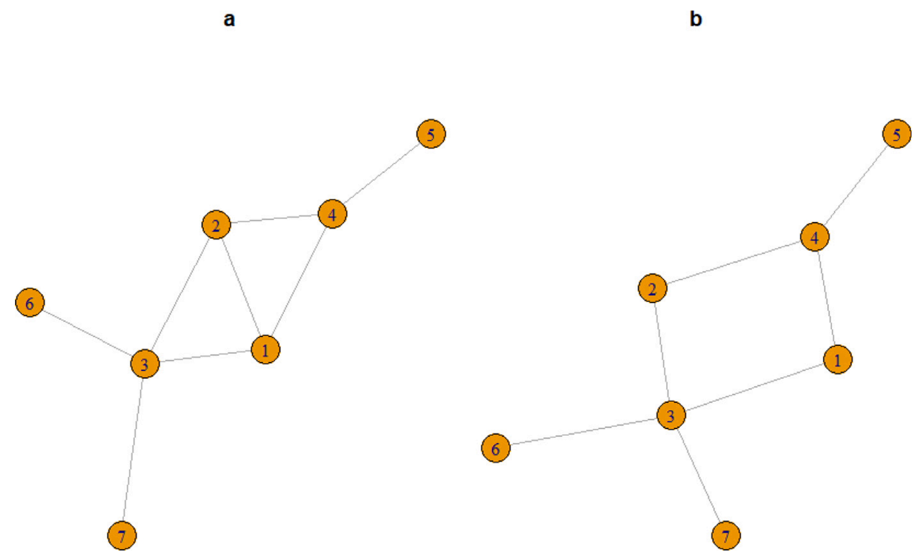

Fig. E.1. Examples of network operation.

There are 7 traders, labeled by $1, 2, 3, \dots, 7$. Each trader observes an initial private information signal $\tilde{\tau}_i = \tilde{u}+\tilde{\epsilon}_i$, where $\tilde{u} \sim N(\bar{u}, h^2)$ represents the return of the risky asset, and $\tilde{\epsilon}_i \sim N(0, s^2)$ represents the noisy part of each signal and its role is to prevent traders from knowing the true value of risky asset.

We define R_i as the neighbor set of trader i. $R_a\{i\}$ is the neighbor set of trader i in Fig. E.1(a), and this set includes trader i himself. We can write the neighbor set of each trader in Fig. E.1(a) as: $R_a\{1\} = \{1, 2, 3, 4\}$; $R_a\{2\} = \{1, 2, 3, 4\}$; $R_a\{3\} = \{1, 2, 3, 6, 7\}$; $R_a\{4\} = \{1, 2, 4, 5\}$; $R_a\{5\} = \{4, 5\}$; $R_a\{6\} = \{3, 6\}$; $R_a\{7\} = \{3, 7\}$. Similarly, we write the neighbor set of each trader in Fig. E.1(b) as: $R_b\{1\} = \{1, 3, 4\}$; $R_b\{2\} = \{2, 3, 4\}$; $R_b\{3\} = \{1, 2, 3, 6, 7\}$; $R_b\{4\} = \{1, 2, 4, 5\}$; $R_b\{5\} = \{4, 5\}$; $R_b\{6\} = \{3, 6\}$; $R_b\{7\} = \{3, 7\}$.

Next, we will explain the four hypotheses one by one as follows.

(i). Traders with more neighbors receive more precise signals;

We take traders 2 and 7 as an example. Trader 2 has 4 neighbors ($R_a\{2\} = \{1, 2, 3, 4\}$), trader 7 has 2 neighbors ($R_a\{7\} = \{3, 7\}$).

Based on the linear structure in information network, after communicating with their neighbors, the new private information of traders 2 and 7 can be written as:

$$\tilde{x}_2 = \frac{\tilde{\tau}_1 + \tilde{\tau}_2 + \tilde{\tau}_3 + \tilde{\tau}_4}{4} = \tilde{u} + \frac{\tilde{\epsilon}_1 + \tilde{\epsilon}_2 + \tilde{\epsilon}_3 + \tilde{\epsilon}_4}{4} = \tilde{u} + \tilde{\eta}_2,$$
$$\tilde{x}_7 = \frac{\tilde{\tau}_3 + \tilde{\tau}_7}{2} = \tilde{u} + \frac{\tilde{\epsilon}_3 + \tilde{\epsilon}_7}{2} = \tilde{u} + \tilde{\eta}_7.$$

where $\tilde{\eta}_2 = \frac{\tilde{\epsilon}_1+\tilde{\epsilon}_2+\tilde{\epsilon}_3+\tilde{\epsilon}_4}{4}$ and $\tilde{\eta}_7 = \frac{\tilde{\epsilon}_3+\tilde{\epsilon}_7}{2}$. Combined with the distribution properties of $\tilde{\epsilon}_i$ ($\tilde{\epsilon}_1 \perp \tilde{\epsilon}_2 \perp \tilde{\epsilon}_3 \perp \tilde{\epsilon}_4 \perp \tilde{\epsilon}_5 \perp \tilde{\epsilon}_6 \perp \tilde{\epsilon}_7$, and $\tilde{\epsilon}_i \sim N(0, s^2)$), we have $D(\tilde{\eta}_2) = \frac{s^2}{4}$; $D(\tilde{\eta}_7) = \frac{s^2}{2}$.

The precision of information is the inverse of its variance. We have $presion(\tilde{\eta}_2) = \frac{4}{s^2} > presion(\tilde{\eta}_7) = \frac{2}{s^2}$, which is the implication of hypothesis (i). In reality, the hypothesis is also reasonable, if one has more neighbor, he will receive more information from others and attain a more precision information.

(ii). All else equal, connected traders have higher signal correlation than non-connected ones;

To illustrate the hypothesis, we define another network in Fig. E.1(b). The structures of network a and b are identical except for the missing link between traders 1 and 2 in network b. We can calculate the correlation between a and b as follows:

In Fig. E.1(a): $Corr_a(\tilde{x}_1, \tilde{x}_2) = Corr(\tilde{u} + \frac{\tilde{\epsilon}_1+\tilde{\epsilon}_2+\tilde{\epsilon}_3+\tilde{\epsilon}_4}{4}, \tilde{u} + \frac{\tilde{\epsilon}_1+\tilde{\epsilon}_2+\tilde{\epsilon}_3+\tilde{\epsilon}_4}{4}) = 1$,

In Fig. E.1(b): $Corr_b(\tilde{x}_1, \tilde{x}_2) = Corr(\tilde{u} + \frac{\tilde{\epsilon}_2+\tilde{\epsilon}_3+\tilde{\epsilon}_4}{4}, \tilde{u} + \frac{\tilde{\epsilon}_1+\tilde{\epsilon}_3+\tilde{\epsilon}_4}{4}) = \frac{h^2+\frac{2s^2}{9}}{h^2+\frac{s^2}{3}}$.

$Corr_b(\tilde{x}_1, \tilde{x}_2) < Corr_a(\tilde{x}_1, \tilde{x}_2)$, which is the economic meanings of hypothesis (ii).

(iii). If two traders have no common neighbors, then the error terms of their signals are uncorrelated;

We take traders 5 and 7 in Fig. E.1(a) as an example. In Fig. E.1(a), $R_a\{5\} = \{4, 5\}$; $R_a\{7\} = \{3, 7\}$, the two traders do not have common neighbors. Under the linear structure, the new private information of traders 5 and 7 can be written as:

$$\tilde{x}_5 = \frac{\tilde{\tau}_4 + \tilde{\tau}_5}{2} = \tilde{u} + \frac{\tilde{\epsilon}_4 + \tilde{\epsilon}_5}{2} = \tilde{u} + \tilde{\eta}_5,$$
$$\tilde{x}_7 = \frac{\tilde{\tau}_3 + \tilde{\tau}_7}{2} = \tilde{u} + \frac{\tilde{\epsilon}_3 + \tilde{\epsilon}_7}{2} = \tilde{u} + \tilde{\eta}_7.$$

The independence of error terms is $(\tilde{\epsilon}_4 \perp \tilde{\epsilon}_5 \perp \tilde{\epsilon}_3 \perp \tilde{\epsilon}_7)$

$$Corr_a(\tilde{\eta}_5, \tilde{\eta}_7) = Corr_a(\frac{\tilde{\epsilon}_4 + \tilde{\epsilon}_5}{2}, \frac{\tilde{\epsilon}_3 + \tilde{\epsilon}_7}{2}) = 0.$$

Thus, if two traders have no common neighbors, then the error terms of their signals are uncorrelated.

(iv). Traders who have the same neighbors receive the same signals.

We take traders 1 and 2 in Fig. E.1(a) as an example. In Fig. E.1(a), $R_a\{1\} = \{1,2,3,4\}$; $R_a\{2\} = \{1,2,3,4\}$, trader 1 and trader 2 have the same neighbor set ($R_a\{1\} = R_a\{2\} = \{1,2,3,4\}$). Under the linear structure, The new private information from traders 1 and 2 can be written as:

$$\tilde{x}_1 = \frac{\tilde{\tau}_1 + \tilde{\tau}_2 + \tilde{\tau}_3 + \tilde{\tau}_4}{4} = \tilde{u} + \frac{\tilde{\epsilon}_1 + \tilde{\epsilon}_2 + \tilde{\epsilon}_3 + \tilde{\epsilon}_4}{4} = \tilde{u} + \tilde{\eta}_1,$$
$$\tilde{x}_2 = \frac{\tilde{\tau}_1 + \tilde{\tau}_2 + \tilde{\tau}_3 + \tilde{\tau}_4}{4} = \tilde{u} + \frac{\tilde{\epsilon}_1 + \tilde{\epsilon}_2 + \tilde{\epsilon}_3 + \tilde{\epsilon}_4}{4} = \tilde{u} + \tilde{\eta}_2.$$

It is obvious that $\tilde{x}_1 = \tilde{x}_2$, which means trader 1 and trader 2 receive the same signals.

The covariance matrix $\mathbf{S} = s^2\mathbf{D}^{-1}\mathbf{W}\mathbf{D}^{-1}$ has the following expanded form:

$$\begin{bmatrix} \frac{1}{[\mathbf{W}]_{11}} & \frac{[\mathbf{W}]_{12}}{[\mathbf{W}]_{11}[\mathbf{W}]_{22}} & \cdots & \frac{[\mathbf{W}]_{1n}}{[\mathbf{W}]_{11}[\mathbf{W}]_{nn}} \\ \frac{[\mathbf{W}]_{21}}{[\mathbf{W}]_{22}[\mathbf{W}]_{11}} & \frac{1}{[\mathbf{W}]_{22}} & \cdots & \frac{[\mathbf{W}]_{2n}}{[\mathbf{W}]_{22}[\mathbf{W}]_{nn}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{[\mathbf{W}]_{n1}}{[\mathbf{W}]_{nn}[\mathbf{W}]_{11}} & \frac{[\mathbf{W}]_{n2}}{[\mathbf{W}]_{nn}[\mathbf{W}]_{22}} & \cdots & \frac{1}{[\mathbf{W}]_{nn}} \end{bmatrix}.$$

The non-diagonal elements measure the correlation of different information after communication via network. The correlation between two traders' information depends on the number of common neighbors ($[\mathbf{W}]_{ij}$). Specifically, if traders do not have common neighbors ($[\mathbf{W}]_{ij} = 0$), there will be no correlation between their information.

References

Bond, P., Goldstein, I., 2015. Government intervention and information aggregation by price. J. Finance 70 (6), 2777–2812.
Chen, S., Li, J., 2021. Can fund networks improve investment performance? J. Financial Res. 6, 170–188, (in chinese).
Cici, G., Jaspersen, S., Kempf, A., 2017. Speed of information diffusion within fund families. Rev. Asset Pricing Stud. 7 (1), 144–170.
Cohen, L., Frazzini, A., Malloy, C., 2008. The small world of investing: Board connections and mutual fund returns. J. Polit. Econ. 116 (5), 951–979.
Colla, P., Mele, A., 2010. Information linkages and correlated trading. Rev. Financ. Stud. 23 (1), 203–246.
Diamond, D.W., 1985. Optimal release of information by firms. J. Finance 40 (4), 1071–1094.
Diamond, D.W., Verrecchia, R.E., 1991. Disclosure, liquidity, and the cost of capital. J. Finance 46 (4), 1325–1359.
Ding, M., Nilsson, B., Suardi, S., 2017. Foreign institutional investment, ownership, and liquidity: real and informational frictions. Financial Rev. 52 (1), 101–144.
Dugast, J., Foucault, T., 2018. Data abundance and asset price informativeness. J. Financ. Econ. 130 (2), 367–391.
Easley, D., Hara, M., 2004. Information and the cost of capital. J. Finance 59 (4), 1553–1583.
El-Khatib, R., Jandik, D., Jandik, T., 2021. Network centrality, connections, and social capital: evidence from CEO insider trading gains. Financial Rev. 56 (3), 433–457.
Frenkel, S., Guttman, I., Kremer, I., 2020. The effect of exogenous information on voluntary disclosure and market quality. J. Financ. Econom. 138, 176–192.
Gao, P., Liang, P.J., 2013. Informational feedback, adverse selection, and optimal disclosure policy. J. Account. Res. 51 (5), 1133–1158.
Goldstein, I., Yang, L., 2017. Information disclosure in financial markets. Annu. Rev. Finan. Econ. 9, 101–125.
Goldstein, I., Yang, L.Y., 2019. Good disclosure, bad disclosure. J. Financial Econ. 131, 118–138.
Grossman, S., 1978. Futher results on the informational efficiency of competitive stock markets. J. Econom. Theory 18 (1), 81–101.
Grossman, S., Stiglitz, J., 1980. On the impossibility of informationally efficient markets. Am. Econ. Rev. 70 (3), 393–408.
Han, B., Tang, Y., Yang, L., 2016. Public information and uninformed trading: Implications for market liquidity and price efficiency. J. Econom. Theory 163, 604–643.
Han, B., Yang, L., 2013. Social networks, information acquisition, and asset prices. Manage. Sci. 59 (6), 1444–1457.
Hara, M., Ye, M., 2011. Is market fragmentation harming market quality? J. Financ. Econ. 100 (3), 459–474.
Hellwig, M.F., 1980. On the aggregation of information in competitive markets. J. Econom. Theory 22 (3), 477–498.
Holthausen, R.W., Verrecchia, R.E., 1988. The effect of sequential information releases on the variance of price changes in an intertemporal multi-asset market. J. Account. Res. 26 (1), 82–106.
Jain, P.C., 1988. Response of hourly stock prices and trading volume to economic news. J. Bus. 61 (2), 219–231.
Kendall, C., 2018. The time cost of information in financial markets. J. Econom. Theory 176, 118–157.
Kim, O., Verrecchia, R.E., 1991. Trading volume and price reactions to public announcements. J. Account. Res. 29 (2), 302–321.
Kyle, A., 1985. Continuous auctions and insider trading. Econometrica 53 (6), 1315–1335.
Lambert, R., Leuz, C., Verrecchia, R.E., 2007. Accounting information disclosure, and the cost of capital. J. Account. Res. 45 (2), 385–420.
Landsman, W.R., Maydew, E.L., Thornock, J.R., 2012. The information content of annual earnings announcements and mandatory adoption of IFRS. J. Account. Econ. 53 (1–2), 34–54.
Lu, R., Li, J., Chen, S., 2022. Portraits of investors' selling behavior in China's stock market: Advances in disposition effect. Manage. World 38 (03), 59–78, (in chinese).
Mondria, X., Yang, L., 2022. Costly interpretation of asset prices. Manage. Sci. 68 (1), 52–74.
Ozsoylev, H.N., Walden, J., 2011. Asset pricing in large information networks. J. Econom. Theory 146, 2252–2280.
Savor, P.G., 2012. Stock returns after major price shocks: The impact of information. J. Financ. Econ. 106 (3), 635–659.
Tetlock, P.C., 2010. Does public financial news resolve asymmetric information. Rev. Financ. Stud. 23 (9), 3520–3557.
Verrecchia, R.E., 1982. Information acquisition in a noisy rational expectations economy. Econometrica 50 (6), 1415–1430.
Vives, X., 2008. Information and Learning in Markets: The Impact of Market Microstructure. Princeton University Press, Princeton, Oxford.
Walden, J., 2019. Trading, profits, and volatility in A dynamic information network model. Rev. Econ. Stud. 86 (5), 2248–2283.
Wang, X., Guo, Q.Y., Luo, R.H., Zhou, J., 2018. Research on stock holdings of institutional investor and market reaction to earning announcement based on information network. China Soft Sci. Manage. 11, 172–183, (in chinese).
Wu, F., 2013. Do stock prices underreact to information conveyed by investors' traders? Evidence from China. Asia-Pacific J. Financial Stud. 42 (3), 442–466.
Xue, H., Zheng, R.H., 2021. Word-of-mouth communication, noise-driven volatility, and public disclosure. J. Account. Econ. 71 (1), 101363.